Heat Transfer of nano fluids in an inclined channel under Magnetic field with heat source/sink

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Abstract:
We investigated theoretically the MHD flow of a nano-fluid past an inclined, oscillatory and permeable semi-infinite flat sinking sheet (S<0). The constant heat source and the thermal radiation are also considered. The slip velocity is oscillatory on time. The governing equations of the boundary layer are solved numerically using the R-K 6^{th} order method. The influence of various parameters on velocity and heat transfer is analyzed. The increase in inclination of the sinking sheet, thinning the momentum boundary and enhancing the thermal boundary layers.

Key words: **Nano - fluid, MHD, Inclined sinking sheet, Radiation, R-K 6^{th} order method.**

1.1 Introduction:

Research in the field of Heat transfer challenging the cooling of many systems used in day to day life of mankind. The heat transfer enhances enormously when nano-particles are suspended in liquids like water, ethylene glycol etc. This has substantiated by Das, Choi and Patel [8] in their review paper. In this scenario cooling systems demand the very low heat transfer rate through nano – fluids and heat energy systems like automobiles demanding the high heat transfer rate through nano – fluids. Akyildiz and Siginer [2] presented analytical solutions for the velocity and temperature fields in a viscous fluid flowing over a nonlinearly stretching sheet by the Galerkin Legendre spectral Method. Prasad et al. [7] presented a numerical solution for the steady two-dimensional mixed convection MHD flow of an electrically conducting viscous fluid over a vertical stretching sheet in its own plane. The stretching velocity and the transverse magnetic field were assumed to vary as a power function of the distance from the origin. Afzal [1] studied the laminar boundary layer flow over a nonlinearly stretching two-dimensional sheet, or axisymmetric plane or the body of revolution arising from nonlinear power law stretching velocity.

Kuznetsov and Nield [5] studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid (Newtonian fluid) past a vertical flat plate to the case of nano-fluids. In these papers the authors have used the nano-fluid model proposed by Buongiorno [3]. Although this author discovered that seven slip mechanisms take place in the convective transport in nano-fluids, it is only the Brownian diffusion and the thermophoresis that are the most important when the turbulent flow effects are absent. More recently, Khan and Aziz [4] studied Natural convection flow of a nano-fluid over a vertical plate with uniform surface heat flux. M. A. A. Hamad and I. Pop [6] presented in their recent paper that the solid volume and heat source enhances the heat transfer rate. This brief survey clearly indicates that a definitive conclusion regarding the role of nano-particles in enhancing natural convective transport is yet to be reached.

In this paper we aim to investigate the MHD Cu – water nano-fluid flow and the heat transfer past a vertical infinite permeable inclined oscillating sinking sheet under heat source, injection (S<0), radiation and magnetic field.

1.2 Mathematical Formulation:

Consider the unsteady free convection flow of a nano-fluid past a vertical permeable semi-infinite sinking sheet in the presence of an applied magnetic field with constant heat source, radiation and suction. We consider a Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$, the flow is assumed to be in the $\bar{x}$ direction, which is taken along the sheet, and $\bar{z}$ - axis is normal to the sheet. We assume that the sheet has an oscillatory movement on time $\bar{t}$ and frequency $\bar{n}$ with the velocity u(0,t),which is given u(0,t) = U_0 (1 + x + ε cos (nt)), where ε is a small constant parameter (ε << 1), x is the rate of sinking and U_0 is the characteristic velocity. We consider that initially (t < 0) the fluid as well as the sheet is at rest. A uniform external magnetic field B_0 is taken to be acting along the $\bar{z}$ -axis. Also assume that the induced magnetic field is small compared to the external magnetic field B_0. The surface temperature is assumed to have the constant value T_w while the ambient temperature has the constant value T_∞, where $T_w > T_\infty$. The conservation equation of current density $\nabla . J = 0$ gives J_z = constant. Since the sheet is electrically non-conducting, this constant is zero. It is assumed that the sheet is infinite in extent and hence all physical quantities do not depend on $\bar{x}$ and $\bar{y}$ but depend only on $\bar{z}$ and $\bar{t}$,

i.e $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Bossinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial w}{\partial z} = 0 \quad \text{.................................... (1)}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial z^2} + (\rho\beta)_{nf}\, g\,(T - T_\infty)\, \text{Cos}\,\gamma - \sigma B_o^2 u\right] \quad \text{.................. (2)}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \alpha_{nf}\frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}}(T - T_\infty) - \frac{\alpha_{nf}}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial z} \quad \text{...................... (3)}$$

The appropriate initial and boundary conditions for the problem are given by

$$\left.\begin{aligned} & u(z,t) = 0, T = T_\infty \text{ for } t < 0\ \forall\ z \\ & u(0,t) = U_0\left[1 + x + \frac{\varepsilon}{2}(e^{int} + e^{-int})\right], T(0,t) = T_w \\ & u(\infty,t) \to 0, T(\infty,t) \to T_\infty,\ \varepsilon << 1 \end{aligned}\right\} \quad t \geq 0 \text{ .... (4)}$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_f + \phi(\rho c_p)_s,$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$k_{nf} = k_f \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \right] \quad \text{........................ (5)}$$

The thermo-physical properties (values) of the materials used are as follows.

Table 1:

Physical Properties	Water	Copper(Cu)
$C_p\ (J/kg\ K)$	4,179	**385**
$\rho\ \ (kg/m^3)$	997.1	**8,933**
$\kappa\ \ (W/m\ K)$	0.613	**400**
$\beta \times 10^{-5}\ (1/K)$	**21**	**1.67**

We consider the solution of Esq. (1) as $w = -w_0$ .... (6)
Where the constant w_0 represents the normal velocity at the sinking sheet which is negative for injection (w_0<0). Thus, we introduce the following dimensionless variables:

$$z = \left(\frac{\nu_f}{U_0}\right) Z,\ t = \left(\frac{\nu_f}{U_0}\right) t^*,\ n = \left(\frac{U_0{}^2}{\nu_f}\right) \eta,\ u = x U U_0,\ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$q_r = -\frac{4\sigma_1}{3\delta} \frac{\partial T^4}{\partial y} \quad \text{......... (7)}$$

We assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty{}^3 - 3T_\infty{}^4$$

By using above, we obtain $\frac{\partial q_r}{\partial z} = -\frac{16\sigma_1}{3\delta} \frac{\partial^2 T^4}{\partial z^2}$ .... (8)

Using equations 5, 6, 7 & 8 the Eqs. 2–3 can be written in the following dimensionless form:

$$\left[1 - \phi + \phi\left(\frac{\rho_s}{\rho_f}\right)\right]\left(\frac{\partial U}{\partial \tau} - S\frac{\partial U}{\partial Z}\right) = \frac{1}{(1-\phi)^{2.5}} \frac{\partial^2 U}{\partial Z^2} + \left[1 - \phi + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right] \theta\ \mathrm{Cos}\,\gamma\ \frac{1}{x} - MU$$

$$\left[1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]\left(\frac{\partial \theta}{\partial \tau} - S\frac{\partial \theta}{\partial Z}\right) = \frac{1}{Pr}\left[\frac{k_{nf}}{k_f}\frac{\partial^2 \theta}{\partial Z^2}\right] - \frac{1}{Pr} Q_H \theta + \frac{1}{Pr}\frac{4}{3} Ra \frac{\partial^2 \theta}{\partial Z^2}$$

Where the corresponding boundary conditions (4) can be written in the dimensionless form as:

$$\left.\begin{aligned} &U(z,t) = 0,\ \theta(z,t) = 0 \ \text{ for } t < 0 \ \forall\ z \\ &U(0,t) = \left[1 + x + \frac{\varepsilon}{2}(e^{int} + e^{-int})\right],\ \theta(0,t) = 1 \\ &U(\infty,t) \to 0,\ \ \theta(\infty,t) \to 0 \end{aligned}\right\} \forall\ t \ge 0$$

Here Pr is the Prandtl number, S is the injection (S < 0) parameter, M is the magnetic parameter, Ra is the Radiation parameter and Q_H is the heat source parameter, which are defined as:

$$Pr = \frac{\nu_f}{\alpha_f},\ S = \frac{w_0}{U_0},\ M = \frac{\sigma B_0^2 \nu_f}{\rho_f U_0^2},\ Ra = \frac{4\alpha\sigma_1 T_\infty^3}{\delta k_{nf}},\ Q_H = \frac{Q \nu_f^2}{k_f U_0^2}$$

Where the velocity characteristic U_0 is defined as

$$U_0 = \left[g\, \beta_f\, (T_w - T_\infty)\, \nu_f\right]^{1/3}$$

The local Nusselt number Nu in dimension less form:

$$Nu = -\, x\, \frac{k_{nf}}{k_f}\, \theta'(0)$$

1.3 Solution of the Problem:

The semi-infinite plate length is limited to 6 for computations. R-K 6th order with shooting method is adopted to solve the governing equations numerically. The convergences of the method are guaranteed by satisfaction of the boundary conditions. The Mathematica package has been used to find the solution numerically.

1.4 Results and Discussions:

The effect of various parameters viz. solid volume fraction (φ), Magnetic parameter (M), heat source parameter (Q_H), Radiation parameter (Ra), suction parameter (S), Sinking parameter (x), inclination angle of the plate (γ) on velocity (U) and temperature (θ) are exhibited in graphs from Figures 1 to 14. The other parameters were assumed constant. The Prandtl Number (Pr) kept constant as 7 (for water), ε = 0.02 and nt = π/2 and the Heat Transfer rate (Nu) is exhibited in Table – 2.

The velocity profiles are shown from Figs. 1 to 7.The momentum decreases with increase in volume fraction (φ) from Fig.1. The 5% of solid volume fraction significantly decreases the momentum when compared with usual fluid. From Fig. 2 As the injection increases, the momentum is also increases (S). From Fig. 3 the magnetic field (M) affects the momentum very much. The momentum is almost linear in the absence of the magnetic field and decreases with increase in M. From Fig. 4 the momentum decreases slightly with increase in inclination angle (γ). From Fig. 5 the heat source (Q_H) does not show much impact on momentum but, the momentum decreases with increase in heat source. From Fig. 6 it shows the increase in radiation (Ra) enhances the momentum. Fig. 7 shows the variation of velocity with sinking parameter (x).

The increase in x retards the fluid leading to enhancement in momentum.

The temperature profiles for various parameters are shown from Fig. 8 to Fig. 14. The variation of temperature is observed near the base of the sheet for all parameters. The temperature slightly increases with solid volume fraction (φ) from Fig. 8. The profile clearly shows the importance of nano-fluid. The variation is low due to the clustering of the metal particles. The increase in injection (S) enhances the thermal boundary layer in Fig. 9. Fig. 10 shows the variation of temperature with magnetic parameter. Temperature decreases with increase in Lorentz force. From Fig. 11 the inclination angle (γ) of the plate enhances the temperature. From Fig. 12 the heat source (Q_H) increases the temperature. From Fig. 13 it shows the increase in radiation increase the temperature. Fig. 14 shows the variation of temperature with x. The increase in sinking retards the flow leading to enhancement of the temperature.

The rate of heat transfer (Nu) for various values of Ra, φ and Q_H are given in the Table-2. The Nu is increasing with increase in φ and Q_H. But Nu decreases with increase in Ra.

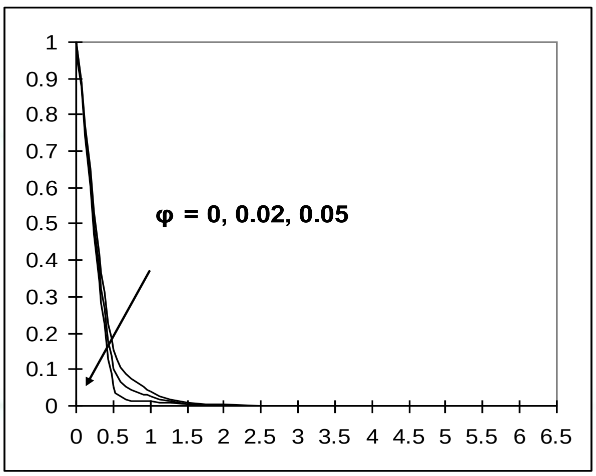


Fig.1 Variation of U with ϕ

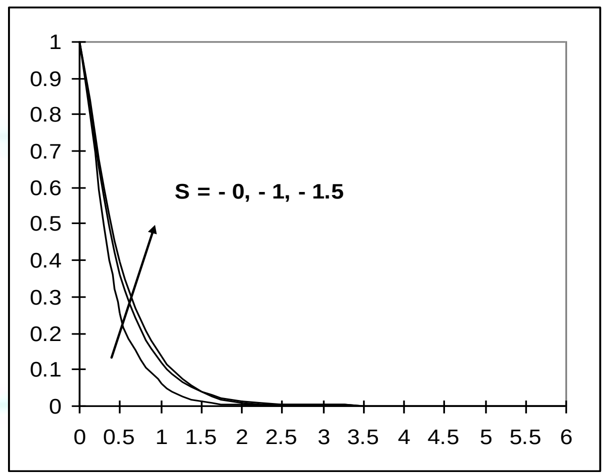


Fig.2 Variation of U with S

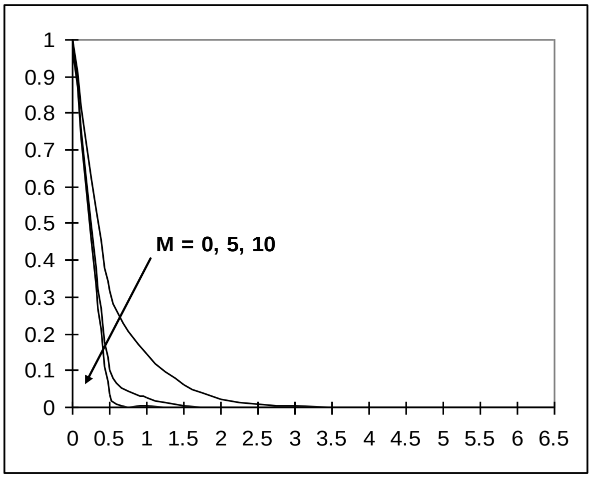


Fig.3 Variation of U with M

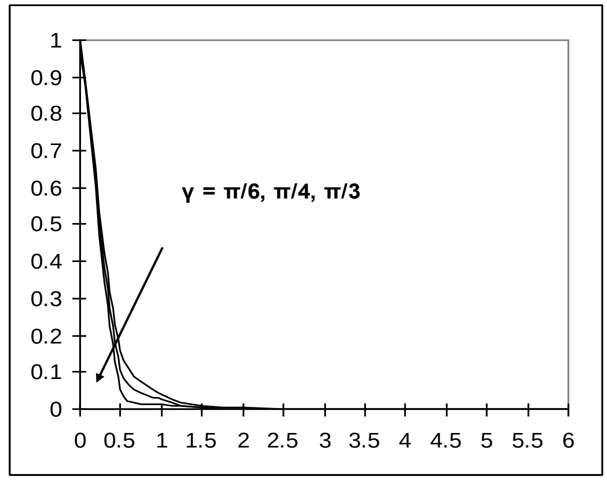


Fig.4 Variation of U with γ

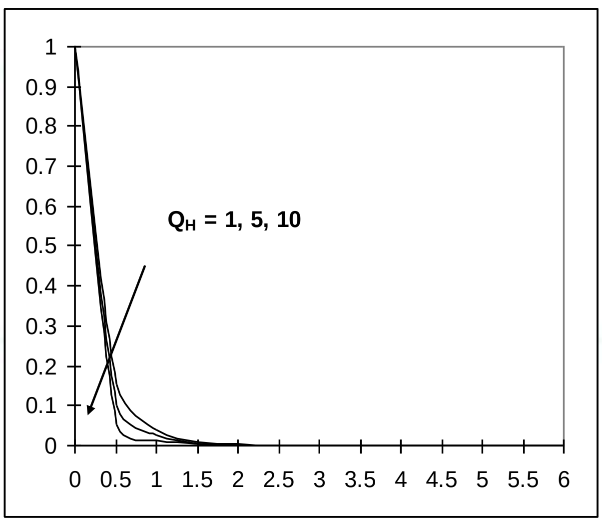


Fig.5 Variation of U with Q_H

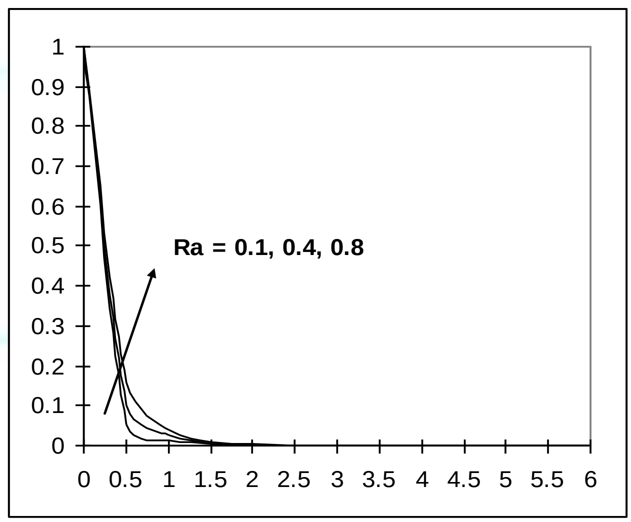


Fig.6 Variation of U with Ra

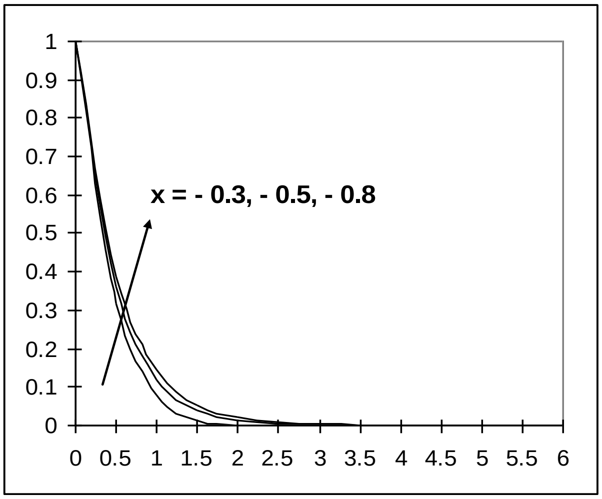


Fig.7 Variation of U with x

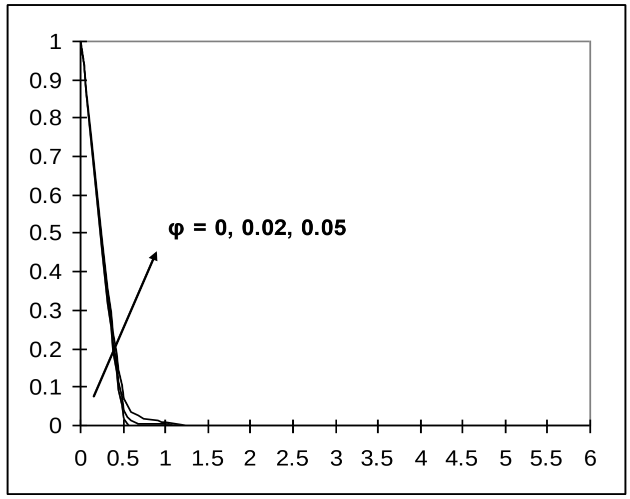


Fig.8 Variation of θ with ϕ

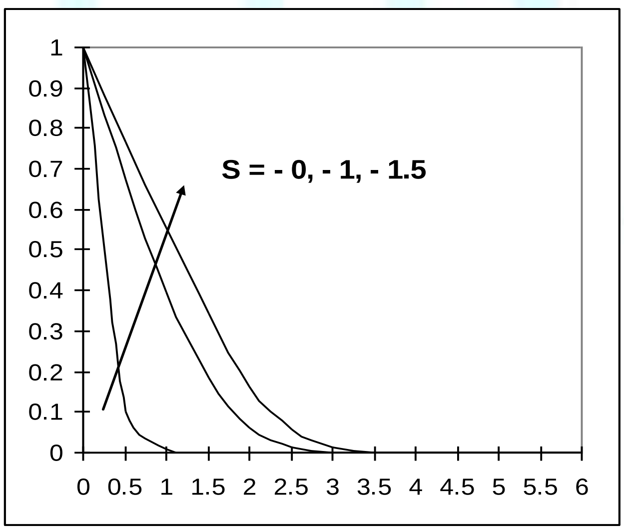


Fig.9 Variation of θ with S

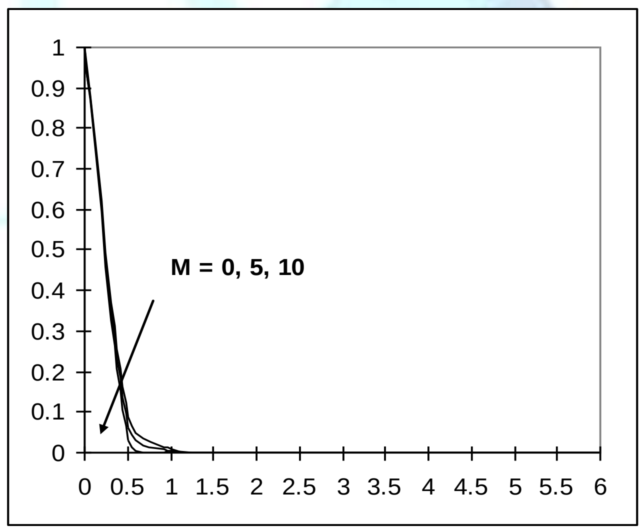
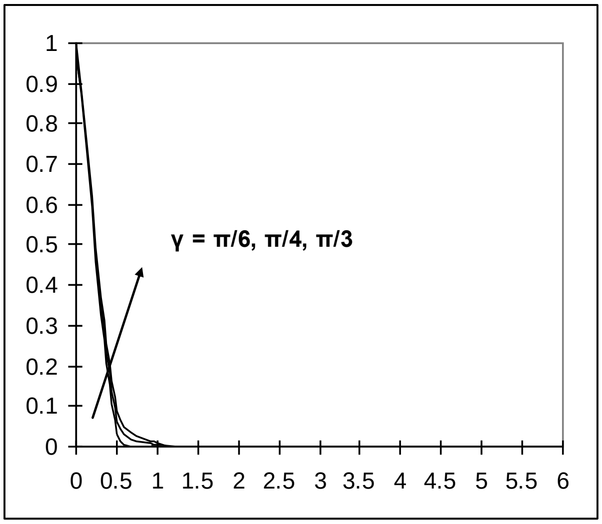


Fig.10 Variation of θ with M



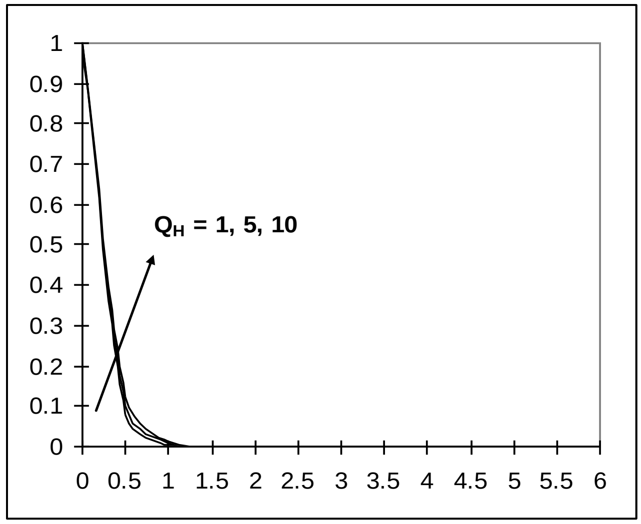
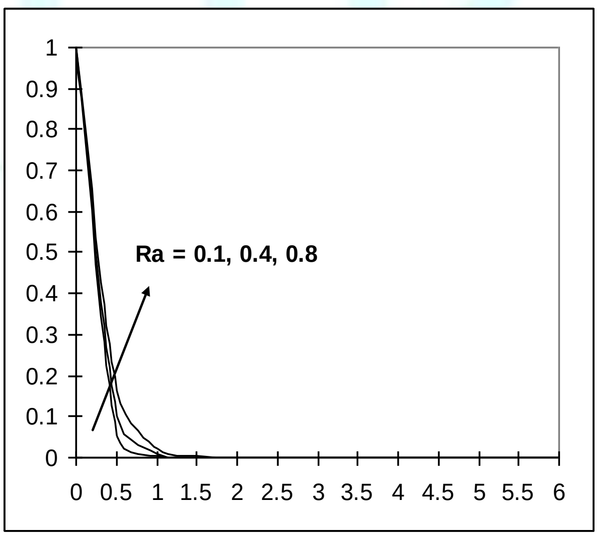


Fig.11 Variation of θ with γ **Fig.12 Variation of θ with Q_H**



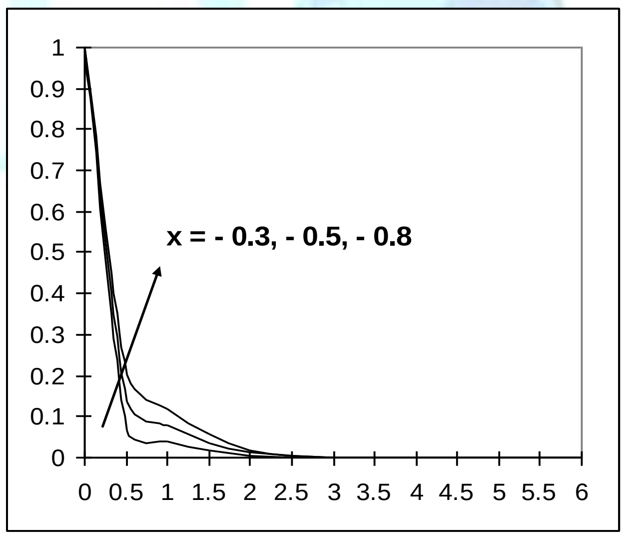


Fig.13 Variation of θ with Ra **Fig.14 Variation of θ with x**

Table-2: Nusselt Number

ϕ	$Q_H = 5;\ Ra = 0.4$	$Q_H = 10;\ Ra = 0.4$	$Q_H = 5;\ Ra = 0.8$

0	4. 5034	4. 8933	**3. 6028**
0.05	4. 9525	5. 4462	**4. 0485**
0.15	5. 8815	6. 6397	**5. 0378**
0.2	**6. 3913**	**7. 3178**	**5. 6092**

1.5 References:

1 Afzal, N. Momentum and thermal boundary layers over a two dimensional or axisymmetric non- linear stretching surface in a stationary fluid. International Journal of Heat and Mass Transfer, 53, 540–547 (2010).

2 Akyildiz, F. T. and Siginer, D. A. Galerkin-Legendre spectral method for the velocity and thermal boundary layers over a non-linearly stretching sheet. Nonlinear Analysis: Real World Applications, 11, 735–741 (2010)

3 Buongiorno J (2006) Convective transport in nano-fluids. ASME J Heat Transfer [128:240–250].

4 Khan, W.A., A. Aziz (2011). Natural convection flow of a nano-fluid over a vertical plate with Uniform surface heat flux, International Journal of Thermal Sciences, [50: 1207-1214].

5 Kuznetsov AV, Nield DA (2010) Natural convective boundary layer flow of a nano-fluid past a vertical plate. Int J Therm Sci [288:243–247].

6 M. A. A. Hamad, I. Pop, Unsteady MHD free convection flow past a vertical permeable flat plate In a rotating frame of reference with constant heat source in a nano-fluid, Heat Mass Transfer (2011) [47:1517–1524].

7 Prasad, K. V., Vajravelu, K., and Datti, P. S. Mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. International Journal of Non-Linear Mechanics, 45, 320–330 (2010)

8 Sarit Kumar Das, Stephen U.S. CHOI , Hrishikesh E. Patel, Heat Transfer in Nano-fluids – A Review, Heat Transfer Engineering, [27(10):3–19, 2006].

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