Comparative analyses of two mathematical models obtained from two modeling techniques for prediction of compressive strength of sand-laterite blocks

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Abstract

Scheffe's and Osadebe's modeling techniques were used in formulating two mathematical models for the prediction of compressive strength of sand laterite blocks. The models were tested for adequacy using statistical tools. Comparative analyses were made on the results of predictions from the two models. The student's t-test statistics proved that there is no significant difference between the predicted results of the two models at an α-level of 0.5. The percentage difference between the two model results ranges from a minimum of 0.25% to a maximum of 6.66% which is also insignificant. These comparisons show that both models are adequate and any of them can be used for prediction of the compressive strength of sand -laterite blocks given the mix ratios or vice versa.

Keywords: Sand-laterite blocks, compressive strength, Scheffe's and Osadebe's modeling techniques

1. Introduction

Building blocks are masonry units used mostly as walling materials in construction. They can be made from a wide variety of materials ranging from binder, water, sand, laterite, coarse aggregate, clay to admixtures. The constituent materials determine the type of block and the choice of these materials depends largely on their availability and affordability. Sand-laterite blocks are made of river sand, laterite, water and cement.

In engineering field, blocks belong to the same family with concrete and they are expected to exhibit similar properties. The desired properties can only be achieved by mixing the constituent materials in their right proportions. The compressive strength is one of the engineering properties that can be used as a yardstick in measuring other properties. It shows the best possible strength the unit can reach in perfect conditions.

The general approach to concrete mixture proportioning reveals that time, energy and money are spent in order to get the desired result (Simon et.al., 1997). In order to minimize some of these limitations, an optimization procedure has been proposed. This process involves fitting empirical models to the data for each performance criterion. A mathematical model stands as a mathematical representation of a set of relationship between variables or parameters. The act of constructing or fashioning a model of something or finding a relationship between variables is called modeling. In modeling, each response (concrete property) is expressed as algebraic function of factors (individual component proportions). The mathematical equations (models) are adopted for forecasting or predictions. Prediction takes us into the future for decision making as we examine different responses arising from changes in controlled variables (Nwaogazie, 1999).

A number of improved prediction techniques have been proposed by including empirical or computational modeling and statistical techniques. Computational techniques such as finite element analysis usually have complexities that are prohibiting. Statistical techniques have the advantage that models formulated from them can be used for prediction more quickly than any other technique and they are also simpler to implement in softwares (Mama and Osadebe, 2011). Simultaneous optimization to meet several constraints is also possible with statistical methods. Scheffe's and Osadebe's optimization theories form the basis of this work. The models formulated from these theories will be compared for the prediction of compressive strength of sand-laterite blocks

2. Methodology

Analytical and experimental procedures were adopted in the course of this work.

2.1 Henry Scheffe's optimisation theory

Scheffe's optimisation method is based on simplex lattice design. A simplex lattice can be described as a structural representation of lines joining the atoms of a mixture. This lattice can be used as a mathematical space in model experiments involving mixtures by considering the atoms as the constituent components of the mixture. For instance in normal concrete mixture, the constituent elements are water, cement, fine and coarse aggregates and so normal concrete mixture gives a simplex of four components. Hence the simplex lattice of this four- component mixture is a three- dimensional solid equilateral tetrahedron. A mixture experiment involves mixing various proportions of two or more components to make different compositions of an end product (Aggarwal, 2002). Mixture components are subject to the constraint that the sum of all the components must be equal to one. i.e.

$$\sum X_i = 1 \qquad (1)$$

where q is the number of components of a mixture and i ranges from 1 to q. X_i is the proportion of the ith component in the mixture. This shows that if we assume the mixture to be a unit quantity, then the sum of all the proportions must be unity. As a result, the factor space reduces to a regular (q-1) dimensional simplex. The lattice part of the simplex lattice design shows that points are spaced regularly on the simplex. The degree of the simplex lattice is defined by the degree of the polynomial that may be used to fit the response surface over the simplex.

Scheffe (1958) developed a theory for experiments with mixtures of q-components whose purpose is the empirical prediction of the response to any mixture of the components, when the response depends only on the proportion of the component and not on the total amount. He introduced the (q,m) simplex lattice designs.

In a (q-1) dimensional simplex, (where q represents the number of vertices)

(a) If q=2, we have 2 points of connectivity, giving a straight line simplex lattice (one dimension)

(ii) If q=3, we have a triangular simplex lattice (two dimensions).

(iii) If q=4, we have a tetrahedron simplex lattice (three dimensions)

Considering a whole factor space in design, we will have (q,m)simplex lattice whose properties are defined as follows:

(a) the factor space has uniformly spaced distribution of points.

(b) The proportions used for each factor have m+1 equally spaced values from 0 to 1 i.e. X_{ij} = 0, 1/m, 2/m, 3/m, ……,1 and all possible mixtures with these proportions for each component used.

For instance, if we have (q,2) lattice, that is a second degree polynomial, (m=2), the following levels of each factor must be used:0, ½, and 1 respectively. For (q,3) lattice, that is a third degree polynomial, (m=3) the levels of each factor are: 0, 1/3, 2/3, and 1 respectively. Scheffe showed that the number of points in (q,m) lattice is given by

$$^{q+m-1}C_m = q(q+1)\ldots\ldots\ldots(q+m-1)/m! \qquad (2)$$

This implies that

1. For a (3,2) lattice, the number of points equals 3(3+1)/2! = 6
2. For a (3,3) lattice, the number of points equals 3(3+1)(3+2)/ 3! = 10

3. For a (4,2) lattice, the number of points equals 4(4+1)/ 2! = 10
4. For a (4,3) lattice, the number of points equals 4(4+1)(4+2)/ 3! = 20

The (q,m) simplex lattice designs are characterised by the symmetric arrangements of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region. The polynomial has exactly as many parameters as there are number of points in the associated simplex lattice design.

Scheffe, (1958) introduced canonical polynomials to be used with his simplex lattice designs. These polynomials are obtained by modifying the usual polynomial model in Xi by using the restriction $\sum X_i = 1$. He assumed that a polynomial function of degree n in the q variables X_1, X_2,……. X_q will be called a '(q,n) polynomial', and that it will be of the form

$$Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \sum b_{i_1 i_2 \ldots i_n} X_{i_1} X_{i_2} \ldots X_{i_n} \qquad (3)$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq j \leq k \leq q$, $i \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively)

and b =constant coefficients

In general, the reduced form of Eqn (3) is in the form of Eqn (4) for a mixture with four components (i.e. n = 4) is given by

$$\begin{aligned} Y = b_0 + {} & b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 \\ & + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 \\ & + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{34} X_3 X_4 \\ & + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 \end{aligned} \qquad (4)$$

Multiplying Eqn (1) by b_o gives Eqn (5)

$$b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 = b_0 \tag{5}$$

Multiplying Eqn (1) successively by X_1, X_2, X_3, and X_4 and rearranging gives Eqn (6)

$$\begin{aligned}
X_1^2 &= X_1 - X_1X_2 - X_1X_3 - X_1X_4 \\
X_2^2 &= X_2 - X_1X_2 - X_2X_3 - X_2X_4 \\
X_3^2 &= X_3 - X_1X_3 - X_2X_3 - X_3X_4 \\
X_4^2 &= X_4 - X_1X_4 - X_2X_4 - X_3X_4
\end{aligned} \tag{6}$$

Substituting Eqns (5) and (6) into Eqn (4) and simplifying yields Eqn (7)

$$\begin{aligned}
Y = {} & b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 \\
& + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 \\
& + b_{34}X_3X_4 + b_{11}(X_1 - X_1X_2 - X_1X_3 - X_1X_4) \\
& + b_{22}(X_2 - X_1X_2 - X_2X_3 - X_2X_4) \\
& + b_{33}(X_3 - X_1X_3 - X_2X_3 - X_3X_4) + b_{44}(X_4 - X_1X_4 - X_2X_4 - X_3X_4)
\end{aligned}$$

Rearranging the Eqn, we have,

$$\begin{aligned}
Y = {} & (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 \\
& + (b_0 + b_4 + b_{44})X_4 + (b_{12} - b_{11} - b_{22})X_1X_2 \\
& + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 \\
& + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 \\
& + (b_{34} - b_{33} - b_{44})X_3X_4
\end{aligned} \tag{7}$$

Let $\alpha_{i} = b_0 + b_i + b_{ii}$ and $\alpha_{ij} = b_{ij} - b_{ii} - b_{ij}$ (8)

Then Eqn (8) becomes

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_{12} X_1 X_2 + \alpha_{13} X_1 X_3 + \alpha_{14} X_1 X_4$$

$$+ \alpha_{23} X_2 X_3 + \alpha_{24} X_2 X_4 + \alpha_{34} X_3 X_4 \qquad (9)$$

The number of coefficients in Eqn (7) has been reduced to 10 in Eqn (9).

In general, Eqn (9) becomes

$$Y = \sum \alpha_i X_i + \sum \alpha_{ij} X_i X_j \qquad (10)$$

where $1 \le i \le q$, $1 \le i \le j \le q$

The unknown coefficients can be determined using

$$\alpha_i = y_i \text{ and } \alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \qquad (11)$$

It is impossible to use the normal mix ratios such as 1:2:4 or 1:3:6 at given water /cement ratio because of the requirement of the simplex that sum of all the components must be one. Hence it is necessary to carry out a transformation from actual to pseudo components. The actual components represent the proportion of the ingredients while the pseudo components represent the proportion of the components of the ith component in the mixture i.e. X_1, X_2, X_3, X_4.

Let X represent pseudo components and Z, actual components. For component transformation we use the following equations:

$$X = BZ \qquad (12)$$

$$Z = AX \qquad (13)$$

where A = matrix whose elements are from the arbitrary mix proportions chosen when Eqn (13) is opened and solved mathematically.

B = the inverse of matrix A

Z = matrix of actual components

X = matrix of pseudo components obtained from the lattice.

Expanding and using Eqns (12) and (13) the actual components Z were determined and presented in Table 1.

Table 1: Pseudo and actual components for Scheffe's (4,2) lattice for sand-laterite blocks

Pseudo Components					Response	Actual Components			
No	X_1	X_2	X_3	X_4		Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.8	1	3.2	4.8
2	0	1	0	0	Y_2	1	1	3.75	8.75
3	0	0	1	0	Y_3	1.28	1	3.334	13.336
4	0	0	0	1	Y_4	2.2	1	2.5	22.5
5	0.5	0.5	0	0	Y_{12}	0.9	1	3.475	6.775
6	0.5	0	0.5	0	Y_{13}	1.04	1	3.267	9.068
7	0.5	0	0	0.5	Y_{14}	1.5	1	2.85	13.65
8	0	0.5	0.5	0	Y_{23}	1.14	1	3.542	11.043
9	0	0.5	0	0.5	Y_{24}	1.6	1	3.125	15.625
10	0	0	0.5	0.5	Y_{34}	1.74	1	2.917	17.918
CONTROL									
11	0.25	0.25	0.5	0	C_1	1.09	1	3.4045	10.0555
12	0.25	0.5	0.25	0	C_2	1.02	1	3.5085	8.909
13	0.67	0.33	0	0	C_3	0.866	1	3.3815	6.1035
14	0	0.67	0.33	0	C_4	1.0924	1	3.6127	10.2634
15	0.3	0.3	0.4	0	C_5	1.052	1	3.4186	9.3994
16	0.2	0.3	0.5	0	C_6	1.1	1	3.432	10.253
17	0.5	0.25	0.25	0	C_7	0.97	1	3.371	7.9215
18	0.25	0.25	0.25	0.25	C_8	1.32	1	3.196	12.3465
19	0	0.25	0.25	0.5	C_9	1.67	1	3.021	16.7715
20	0	0.25	0	0.75	C_{10}	1.9	1	2.8125	19.0625

Legend:

X_1 = Water/cement ratio Z_1 = Actual water/cement ratio

X_2 = Fraction of cement Z_2 = Actual cement quantity

X_3 = Fraction of river sand Z_3 = Actual river sand quantity

X_4 = Fraction of laterite Z_4 = Actual laterite quantity

2.2 Osadebe's regression theory

The formulation of the regression equation is done from first principles using the so-called absolute volume (mass) as a necessary condition. This principle assumes that the volume (mass) of a mixture is equal to the sum of the absolute volume (mass) of all the constituent components.

Osadebe (2003) assumed that the response function, $F(z)$ is continuous and differentiable with respect to its predictors, Zi.

$$F(z) = F(z^{(0)}) + \sum [\partial F(z^{(0)}) / \partial z_i](z_i - z_i^{(0)}) + ½! \sum\sum [\partial^2 F(z^{(0)}) / \partial z_i \partial z_j](z_i - z_i^{(0)})$$

$$(z_j - z_j^{(0)}) + ½! \sum\sum [\partial^2 F(z^{(0)}) / \partial z_i^2] (z_i - z_i^{(0)})^2 + \ldots\ldots \quad (14)$$

where $1 \le i \le 4$, $1 \le i \le 4$, $1 \le j \le 4$,and $1 \le i \le 4$ respectively.

By making use of Taylor's series, the response function could be expanded in the neighbourhood of a chosen point:

$$Z^{(0)} = Z_1^{(0)}, Z_2^{(0)}, Z_3^{(0)}, Z_4^{(0)}, Z_5^{(0)} \quad (15)$$

Without loss of generality of the formulation, the point $z^{(0)}$ will be chosen as the origin for convenience sake. It is worthy of note here that the predictor, z_i is not the actual portion of the mixture component rather it is the ratio of the actual portions to the quantity of concrete. For convenience sake, let z_i be the fractional portion and s_i be the actual portions of the mixture components.

If the total quantity of concrete is designated s, then

$$\sum s_i = s \quad (16)$$

For concrete of four components, $1 \le i \le 4$ and so Eqn (16) becomes:

$$s_1 + s_2 + s_3 + s_4 = s \quad (17)$$

If the total quantity of concrete required is a unit quantity, then Eqn (17) should be divided throughout by s. Hence

$$s_1/s + s_2/s + s_3/s + s_4/s = s/s \quad (18)$$

But, fractional portions, $z_i = s_i/s$ (19)

Substituting Eqn (19) into Eqn (18) gives Eqn (20)

$$z_1 + z_2 + z_3 + z_4 = 1 \quad (20)$$

In the formulation of the regression equation, the point, $z^{(0)}$ was chosen as the origin.

This implies that $z^{(0)} = 0$ and so

$$z_1^{(0)} = 0,\ z_2^{(0)} = 0,\ z_3^{(0)} = 0 \text{ and } z_4^{(0)} = 0$$

Let

$$b_0 = F(0),\ b_i = \partial F(0) / \partial zi,\ bij = \partial^2 F(0) / \partial z_i \partial zj,\ bii = \partial^2 F(0) / \partial z_i^2$$

Eqns (20) can be rewritten as

$$F(z) = b_0 + \sum b_i z_i + \sum\sum b_{ij} z_i z_j + \sum b_{ii} z_i^2 + \ldots \quad (21)$$

where $1 \leq i \leq 4$ and $1 \leq j \leq 4$

Multiplying Eqn (20) by b_0 gives the expression for b_0 i.e. Eqn (22)

$$b_0 = b_0 z_1 + b_0 z_2 + b_0 z_3 + b_0 z_4 \quad (22)$$

Multiplying Eqn (20) successively by z_1, z_2, z_3 and z_4, and rearranging the products, gives respectively, Eqns (23)-(26)

$$z_1^2 = z_1 - z_1 z_2 - z_1 z_3 - z_1 z_4 \quad (23)$$

$$z_2^2 = z_2 - z_1 z_2 - z_2 z_3 - z_2 z_4 \quad (24)$$

$$z_3^2 = z_3 - z_1 z_3 - z_2 z_3 - z_3 z_4 \quad (25)$$

$$z_4^2 = z_4 - z_1 z_4 - z_2 z_4 - z_3 z_4 \quad (26)$$

Substituting Eqns (22) – (26) into Eqn (21) and simplifying yields Eqn (27)

$$Y = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_{12} z_1 z_2 + \alpha_{13} z_1 z_3 + \alpha_{14} z_1 z_4 + \alpha_{23} z_2 z_3$$

$$+ \alpha_{24} z_2 z_4 + \alpha_{34} z_3 z_4 \tag{27}$$

where

$$\alpha_i = b_0 + b_i + b_{ii} \text{ and } \alpha_{ij} = b_{ij} - b_{ii} - b_{jj} \tag{28}$$

In general, Eqn (27) is given as:

$$Y = \sum \alpha_i z_i + \sum \alpha_{ij} z_i z_j \tag{29}$$

where $1 \leq i \leq j \leq 4$

Eqns (27) is the optimization model equation.

Y is the response function at any point of observation, z_i are the predictors and α_i are the coefficients of the optimization model equations.

Different points of observation will have different responses with different predictors at constant coefficients. At the nth observation point, $Y^{(n)}$ will correspond with $Z_i^{(n)}$. That is to say that:

$$Y^{(n)} = \sum \alpha_i z_i^{(n)} + \sum \alpha_{ij} z_i^{(n)} z_j^{(n)} \tag{30}$$

where $1 \leq i \leq j \leq 4$ and n = 1,2,3, …………. 10

Eqn (30) can be put in matrix from as

$$[Y^{(n)}] = [Z^{(n)}] \{\square\} \tag{31}$$

Rearranging Eqn (31) gives:

$$\{\square\} = [Z^{(n)}]^{-1} [Y^{(n)}] \tag{32}$$

The values of the constant coefficients α_i can be determined using Eqn 32.

The actual mix proportions, $s_i^{(n)}$) and the corresponding fractional portions, $z_i^{(n)}$ are presented on Table 2.

Table 2: Values of actual mix proportions and their corresponding fractional portions

N	S_1	S_2	S_3	S_4	RESPONSE	Z_1	Z_2	Z_3	Z_4
1	0.8	1	3.2	4.8	Y_1	0.08163	0.10204	0.32653	0.4898
2	1	1	3.75	8.75	Y_2	0.06897	0.06897	0.25862	0.60345
3	1.28	1	3.334	13.336	Y_3	0.06755	0.05277	0.17594	0.70375
4	2.2	1	2.5	22.5	Y_4	0.07801	0.03546	0.08865	0.79787
5	0.9	1	3.475	6.775	Y_{12}	0.07407	0.0823	0.28601	0.55761
6	1.04	1	3.267	9.068	Y_{13}	0.07235	0.06957	0.22727	0.63082
7	1.5	1	2.85	13.65	Y_{14}	0.07895	0.05263	0.15	0.71842
8	1.14	1	3.542	11.043	Y_{23}	0.06816	0.05979	0.21178	0.66027
9	1.6	1	3.125	15.625	Y_{24}	0.07494	0.04684	0.14637	0.73185
10	1.74	1	2.917	17.918	Y_{34}	0.07381	0.04242	0.12373	0.76004

Table 3: $Z^{(n)}$ matrix

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.08163	0.10204	0.32653	0.4898	0.00833	0.02666	0.03998	0.03332	0.04998	0.15993
0.06897	0.06897	0.25862	0.60345	0.00476	0.01784	0.04162	0.01784	0.04162	0.15606
0.06755	0.05277	0.17594	0.70375	0.00356	0.01188	0.04754	0.00928	0.03714	0.12381
0.07801	0.03546	0.08865	0.79787	0.00277	0.00692	0.06225	0.00314	0.02829	0.07073
0.07407	0.0823	0.28601	0.55761	0.0061	0.02119	0.0413	0.02354	0.04589	0.15948
0.07235	0.06957	0.22727	0.63082	0.00503	0.01644	0.04564	0.01581	0.04388	0.14337
0.07895	0.05263	0.15	0.71842	0.00416	0.01184	0.05672	0.00789	0.03781	0.10776
0.06816	0.05979	0.21178	0.66027	0.00408	0.01444	0.045	0.01266	0.03948	0.13983
0.07494	0.04684	0.14637	0.73185	0.00351	0.01097	0.05485	0.00686	0.03428	0.10712
0.07381	0.04242	0.12373	0.76004	0.00313	0.00913	0.0561	0.00525	0.03224	0.09404

2.3 Experimental procedure

The actual components as were used to measure out the quantities water (Z_1), cement (Z_2), river sand (Z_3) and laterite (Z_4) for sand-laterite blocks in their respective ratios for the various tests. A total of twenty mix ratios were used to produce sixty 450mm x 150mm x 225mm (solid) blocks that were cured and tested on the 28th day. Ten out of the twenty mix ratios were used as control mix ratios to produce thirty blocks for the confirmation of the adequacy of the mixture design model. Three blocks were tested for each point and the average taken as the result of the point. The failure loads were recorded and the compressive strength was obtained using

$$f_c = P/A \tag{33}$$

where f_c is the compressive strength

P = failure load

A = cross-sectional area of the specimen

3. Results and Discussion

The results of the experimental and analytical procedures are shown below.

3.1 Results of experimental procedures

The experimental values of compressive strength of the sand-laterite blocks are presented on Table 4

Table 4: Compressive strength test results of sand-laterite blocks

Exp No.	Mix ratios (w/c: cement: sand: laterite)	Replicat	Mas (kg)	Densit ρ (kg/m	Avera Densit (kg/m	Failure Load (KN)	X-sectio Area (mm^2)	Compress Strength f_c (N/ mm^2)	Average f_{cu} (N/ mm
1	0.8:1:3.2:4.8	A	24.5	1613.1		180		2.667	3.012
		B	24.7	1626.3	1619.7	220	67500	3.259	
		C	24.6	1619.7		210		3.111	
2	1:1:3.75:8.75	A	22.6	1448.0		130		1.926	2.025
		B	25.1	1652.6	1569.2	140	〃	2.074	
		C	23.8	1567.0		140		2.074	
3	1.28:1:3.334:13.336	A	23.0	1514.4		100		1.482	1.630
		B	23.1	1520.9	1516.6	110	〃	1.630	
		C	23.0	1514.4		120		1.778	
4	2.2:1:2.5:22.5	A	23.1	1520.9		80		1.185	1.259
		B	23.5	1547.3	1534.1	90	〃	1.333	
		C	23.3	1534.1		85		1.259	
5	0.9:1:3.475:6.775	A	23.5	1547.3		170		2.519	

		B	23.0	1514.4	1527.5	140	″	2.074	2.321
		C	23.1	1520.9		160		2.370	
6	1.04:1:3.267:9.068	A	23.0	1514.4		100		1.482	2.074
		B	23.1	1520.9	1516.6	180	″	2.667	
		C	23.0	1514.4		140		2.074	
7	1.5:1:2.85:13.65	A	23.3	1534.1		110		1.630	1.704
		B	23.3	1534.1	1527.5	115	″	1.704	
		C	23.0	1514.4		120		1.778	
8	1.14:1:3.542:11.043	A	23.0	1514.4		140		2.074	1.926
		B	23.0	1514.4	1518.7	120	″	1.778	
		C	23.2	1527.5		130		1.926	
9	1.6:1:3.125:15.625	A	24.2	1593.4		60		0.889	1.185
		B	24.5	1613.1	1595.6	90	″	1.333	
		C	24.0	1580.2		90		1.333	
10	1.74:1:2.917:17.918	A	22.0	1448.5		60		0.889	1.235
		B	23.9	1573.6	1512.2	90	″	1.333	
		C	23.0	1514.4		100		1.482	
11	1.09:1:3.4045:10.0555	A	23.9	1573.6		120		1.778	2.024
		B	23.7	1560.4	1560.5	150	″	2.222	
		C	23.5	1547.3		140		2.074	
12	1.02:1:3.5085:8.909	A	24.0	1580.2		140		2.074	1.975
		B	22.6	1487.2	1544.5	130	″	1.926	
		C	23.7	1566.0		130		1.926	
13	0.866:1:3.3815:6.1035	A	23.5	1547.3		170		2.519	2.666
		B	22.5	1481.4	1520.5	220	″	3.259	
		C	23.2	1532.7		150		2.222	
14	1.0924:1:3.6127:10.2634	A	22.2	1463.9		140		2.074	1.926
		B	23.1	1525.3	1487.5	120	″	1.777	
		C	22.3	1473.2		130		1.926	
15	1.052:1:3.4186:9.3994	A	24.3	1600.0		140		2.074	1.975
		B	23.5	1547.3	1578.0	120	″	1.778	
		C	24.1	1586.8		140		2.074	
16	1.1:1:3.432:10.253	A	22.3	1468.0		140		2.074	1.876
		B	23.2	1527.5	1496.1	120	″	1.778	
		C	22.6	1492.8		120		1.778	

17	0.97:1:3.371:7.92 15	A	23.0	1514.4		150		2.222	2.173
		B	23.2	1527.5	1520.9	140	״	2.074	
		C	23.1	1520.9		150		2.222	
18	1.32:1:3.196:12.3 465	A	25.6	1685.6		110		1.630	1.571
		B	23.8	1567.0	1610.9	98	״	1.452	
		C	24.0	1580.2		110		1.630	
19	1.67:1:3.021:16.7 715	A	24.1	1590.6		80		1.185	1.210
		B	22.6	1487.8	1546.9	80	״	1.185	
		C	23.7	1562.3		85		1.259	
20	1.9:1:2.8125:19.0625	A	23.8	1567.0		80		1.185	1.136
		B	23.6	1558.6	1584.3	80	״	1.185	
		C	24.7	1627.2		70		1.037	

The coefficients of Scheffe's regression equation were determined using Eqn (11) and substituted into polynomial equation given by Scheffe (Eqn 9) to yield

$$Y = 3.01X_1 + 2.03X_2 + 1.63X_3 + 1.26X_4 - 0.8X_1X_2 - 1X_1X_3 - 1.74X_1X_4$$

$$+ 0.4X_2X_3 - 1.82X_2X_4 - 0.82X_3X_4 \quad (34)$$

Eqn (34) is the Scheffe's mathematical model for optimisation of compressive strength of sand-laterite block based on 28-day strength.

The coefficients of Osadebe regression equation were determined using Eqn (32) and substituted into polynomial equation given by Osadebe (Eqn 27) to yield

$$Y = -6966.045Z_1 - 14802.675Z_2 - 418.035Z_3 - 27.196Z_4 + 47847.731Z_5 + 1380.941Z_6$$

$$+ 7862.325Z_7 + 20697.830Z_8 + 13162.925Z_9 + 842.339Z_{10} \quad (35)$$

Eqn (35) is the Osadebe's mathematical model for optimisation of compressive strength of sand-laterite block based on 28-day strength.

The two mathematical models express compressive strength as a multivariate function of proportions of its constituent components

The results predicted by Scheffe's and Osadebe's models for the compressive strength of sand-laterite blocks are presented along side with the laboratory results on Tables 5

Table 5: Compressive strength results of sand-laterite blocks from Scheffe's model, Osadebe's model and laboratory investigation

Exp. No	Mix ratios (w/c: cement: sand: laterite)	Laboratory Compressive Strength Results (N/mm^2)	Scheffe's Model Results (N/mm^2)	Osadebe's Model Results (N/mm^2)
1	0.8:1:3.2:4.8	3.012	3.009	3.012
2	1:1:3.75:8.75	2.025	2.030	2.025
3	1.28:1:3.334:13.336	1.630	1.630	1.630
4	2.2:1:2.5:22.5	1.259	1.260	1.259
5	0.9:1:3.475:6.775	2.321	2.320	2.321
6	1.04:1:3.267:9.068	2.074	2.070	2.074
7	1.5:1:2.85:13.65	1.704	1.700	1.704
8	1.14:1:3.542:11.043	1.926	1.930	1.926
9	1.6:1:3.125:15.625	1.185	1.190	1.185
10	1.74:1:2.917:17.918	1.235	1.240	1.235
11	1.09:1:3.4045:10.0555	2.024	1.950	1.985
12	1.02:1:3.5085:8.909	1.975	2.063	2.104
13	0.866:1:3.3815:6.1035	2.666	2.510	2.493
14	1.0924:1:3.6127:10.2634	1.926	1.986	1.991
15	1.052:1:3.4186:9.3994	1.975	2.020	2.058
16	1.1:1:3.432:10.253	1.876	1.938	1.971
17	0.97:1:3.371:7.9215	2.173	2.220	2.239
18	1.32:1:3.196:12.3465	1.571	1.621	1.568
19	1.67:1:3.021:16.7715	1.210	1.240	1.232
20	1.9:1:2.8125:19.0625	1.136	1.111	1.185

3.2 Tests of adequacy

The two model equations were tested separately for adequacy against the controlled experimental results using the Fisher test statistics. Scheffe's model results were also tested for adequacy against Osadebe's model results using the student's t-test statistics. The statistical hypothesis for these mathematical models is as follows:

Null Hypothesis (H_0): There is no significant difference between the experimental and the theoretically expected results at an α-level of 0.5.

There is no significant difference between the two models.

Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected results at an α-level of 0.05.

There is a significant difference between the two models.

3.2.1 Fisher Test

Table 6: F-statistics test computations for Scheffe's compressive strength model

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)} - y_{(obs)}$	$Y_{(pre)}-y_{(pre)}$	$(Y_{(obs)} - y_{(obs)})^2$	$(Y_{(pre)} - y_{(pre)})^2$
C_1	2.024	1.95	0.1708	0.0841	0.029173	0.007073
C_2	1.975	2.063	0.1218	0.1971	0.014835	0.038848
C_3	2.666	2.51	0.8128	0.6441	0.660644	0.414865
C_4	1.926	1.986	0.0728	0.1201	0.0053	0.014424
C_5	1.975	2.02	0.1218	0.1541	0.014835	0.023747
C_6	1.876	1.938	0.0228	0.0721	0.00052	0.005198
C_7	2.173	2.22	0.3198	0.3541	0.102272	0.125387
C_8	1.571	1.621	-0.2822	-0.2449	0.079637	0.059976
C_9	1.21	1.24	-0.6432	-0.6259	0.413706	0.391751
C_{10}	1.136	1.111	-0.7172	-0.7549	0.514376	0.569874
$\sum$	18.532	18.659			1.835298	1.651143
	$Y_{(obs)}$=1.8532	$y_{(pre)}$=1.8659				

Legend: $y = \sum Y/n$

where Y is the response and n the number of responses.

The variance is

$$S^2 = [1/(n-1)][\sum (Y-y)^2] \qquad (36)$$

Using Eqn (36), $S^2_{(obs)}$ and $S^2_{(pre)}$ are calculated as follows:

$S^2_{(obs)}$ = 1.835298/9 = 0.2039 and $S^2_{(pre)}$ = 1.651143/9 = 0.183

The test statistics is given by

$$F = S_1^2 / S_2^2 \qquad (37)$$

where S_1^2 is the larger of the two variances

With reference to Eqn (37), S_1^2 = 0.2039 and S_2^2 = 0.183

Therefore, F = 0.2039/0.183 = 1.114

From standard Fisher table, $F_{0.95}(9,9)$ = 3.25 which is higher than the calculated F-value. Hence the regression equation is adequate.

Table 7: F-statistics test computations for Osadebe's compressive strength model

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)}$ - $y_{(obs)}$	$Y_{(pre)}$-$y_{(pre)}$	$(Y_{(obs)} - y_{(obs)})^2$	$(Y_{(pre)} - y_{(pre)})^2$
C_1	2.024	1.985	0.1708	0.1024	0.029173	0.010486
C_2	1.975	2.104	0.1218	0.2214	0.014835	0.049018
C_3	2.666	2.493	0.8128	0.6104	0.660644	0.372588
C_4	1.926	1.991	0.0728	0.1084	0.0053	0.011751
C_5	1.975	2.058	0.1218	0.1754	0.014835	0.030765
C_6	1.876	1.971	0.0228	0.0884	0.00052	0.007815
C_7	2.173	2.239	0.3198	0.3564	0.102272	0.127021
C_8	1.571	1.568	-0.2822	-0.3146	0.079637	0.098973
C_9	1.21	1.232	-0.6432	-0.6506	0.413706	0.42328
C_{10}	1.136	1.185	-0.7172	-0.6976	0.514376	0.486646
$\sum$	18.532	18.826			1.835298	1.618342
	$y_{(obs)}$=1.8532	$Y_{(pre)}$=1.8826				

Legend: $y = \sum Y / n$

where Y is the response and n the number of responses.

Using Eqn (36), $S^2_{(obs)}$ and $S^2_{(pre)}$ are calculated as follows:

$S^2_{(obs)}$ = 1.835298/9 = 0.2039 and $S^2_{(pre)}$ = 1.618342/9 = 0.1798

With reference to Eqn (37), S_1^2 = 0.2039 and S_2^2 = 0.1798

Therefore, F = 0.2039/0.1798 = 1.134

From standard Fisher table, $F_{0.95}(9,9)$ = 3.25 which is higher than the calculated F-value. Hence the regression equation is adequate.

The fisher test statistics used for this test proved that there is no significant difference between the experimental and the theoretically expected results at an α-level of 0.5.

3.2.2 Student's t-test

Scheffe's model results were tested for adequacy against Osadebe's model results using the student's t-test statistics as follows:

Table 8: T-statistics test computations

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ε	$y_{(observed)}$ Scheffe's	$y_{(predicted)}$ Osadebes	Δ_Y	t
1	C_1	1 1 1 2 2 3 4	2 3 4 3 4 4 -	-0.125 -0.125 -0.125 -0.125 -0.125 0 0	0.25 0.25 0 0.5 0 0 -	0.0156 0.0156 0.0156 0.0156 0.0156 0 0	0.0625 0.25 0 0.25 0 0 0					
					∑	0.0781	0.5625	0.6406	1.950	1.985	0.035	0.064
Similarly												
2		-	-	-	-	-	-	0.625	2.063	2.104	0.041	0.075
3		-	-	-	-	-	-	0.963	2.510	2.493	0.017	0.028
4		-	-	-	-	-	-	0.899	1.986	1.991	0.005	0.008
5		-	-	-	-	-	-	0.669	2.020	2.058	0.038	0.069
6		-	-	-	-	-	-	0.650	1.938	1.971	0.033	0.060
7		-	-	-	-	-	-	0.609	2.220	2.239	0.019	0.035
8		-	-	-	-	-	-	0.484	1.621	1.568	0.053	0.102
9		-	-	-	-	-	-	0.609	1.240	1.232	0.008	0.015
10		-	-	-	-	-	-	0.734	1.111	1.185	0.074	0.131

T-value from table

For a significant level, α = 0.05, $t_{\alpha/l}(v_e) = t_{0.05/10}(9) = t_{0.005}(9)$ = 3.250 (from standard t-table)

This value is greater than any of the t-values obtained by calculation (as shown in Table 8). Therefore, we accept the Null hypothesis. There is no significant difference between the two models.

The student's t-test statistics used for this test proved that there is no significant difference between the two models. So the two models can be used for the prediction of compressive strength of sand-laterite blocks.

3.3 Comparison of strength values predictable by two optimization models

The results from the models for compressive strength test result of sand-laterite blocks (chosen samples) of the controlled points are presented on Table 9

Table 9: Comparison of compressive strength test results of sand-laterite blocks for Scheffe's model and Osadebe's model and percentage difference

ExpNo	Mix ratios (w/c: cement: sand: laterite)	Laboratory Compressive Strength Results (N/mm^2)	Scheffe's Model Compressive Strength Results (N/mm^2)	Osadebe's Model Compressive Strength Results (N/mm^2)	Percentage Difference btw Lab. & Scheffe's Results (%)	Percentage Difference btw Lab & Osadebe's Results (%)	Percentage Difference btw Scheffe's & Osadebe's Model Results(%)
11	1.09:1:3.4045:10.0555	2.024	1.950	1.985	3.66	1.93	1.79
12	1.02:1:3.5085:8.909	1.975	2.063	2.104	4.45	6.53	1.99
13	0.866:1:3.3815:6.1035	2.666	2.510	2.493	5.85	6.49	0.68
14	1.0924:1:3.6127:10.2634	1.926	1.986	1.991	3.12	3.37	0.25
15	1.052:1:3.4186:9.3994	1.975	2.020	2.058	2.28	4.20	1.88
16	1.1:1:3.432:10.253	1.876	1.938	1.971	3.30	5.06	1.70
17	0.97:1:3.371:7.9215	2.173	2.220	2.239	2.16	3.04	0.86
18	1.32:1:3.196:12.3465	1.571	1.621	1.568	3.18	0.19	3.27
19	1.67:1:3.021:16.7715	1.210	1.240	1.232	2.48	1.82	0.65

20	1.9:1:2.8125:19.0625	1.136	1.111	1.185	2.20	4.31	6.66

Table 9 shows compressive strength test results from laboratory/experimental investigation, Scheffe's model and Osadebe's model. A comparison of predicted results from Scheffe's model with the laboratory results shows that the percentage difference ranges from a minimum of 2.16% to a maximum of 5.85% which is insignificant. A comparison of predicted results from Osadebe's model with the laboratory results shows that the percentage difference ranges from a minimum of 0.19% to a maximum of 6.53% which is also insignificant.

The percentage difference between Scheffe's model result and Osadebe's model result (for compressive strength of sand-laterite blocks) ranges from a minimum of 0.25% to a maximum of 6.66% which is insignificant. This comparison shows that both models are adequate and any of them can be used for optimisation of the block properties. However, the following differences can be noted:

(i) Scheffe's model programs can predict the maximum value of any property while Osadebe's model program cannot give the maximum value.

(ii) In formulation of Scheffe's model, the mixture components are subject to the constraint that the sum of all the components must be equal to one whereas Osadebe's model is not subject to this constraint. Consequently, component transformation is required in Scheffe's model development while the transformation is not needed in Osadebe's case. This makes Osadebe's optimization technique easier to apply because it uses actual mix ratios.

(iii) Any value of property higher than the maximum in the case of Scheffe's model program can be specified as input in Osadebe's model program to obtain the mix ratios that can yield that. This is not obtainable with Scheffe's model program.

4. CONCLUSION

1. Two mathematical models for the prediction of compressive strength of sand laterite blocks were formulated using Scheffe's and Osadebe's modeling techniques.

2. The models were found to be adequate using the Fisher test.

3. Comparative analyses were made on the results of predictions from the two models.

4. The student's t-test statistics proved that there is no significant difference between the predicted results of the two models at an α-level of 0.5.

5. The percentage difference between the two model results ranges from a minimum of 0.25% to a maximum of 6.66% which is insignificant.

6. These comparisons show that both models are adequate and any of them can be used for prediction of the compressive strength of sand -laterite blocks given the mix ratios or vice versa.

5. REFERENCES

Aggarwal, M.I., (2002). *Mixture experiments*. Design workshop lecture notes, University of Delhi, Delhi India, 77-89.

Mama, B.O. and Osadebe, N.N.,(2011). Comparative analysis of two mathematical models for prediction of compressive strength of sandcrete blocks using alluvial deposits. *Nigerian Journal of Technology*, Vol. 30(3).

Nwaogazie, I.L., Probability and statistics for science and engineering practice, Prints Konzults, Lagos, 1999.

Osadebe, N.N., (2003). *Generalised mathematical modelling of compressive strength of normal concrete as a multi-variant function of the properties of its constituent components*. A paper delivered at the College of Engineering, University of Nigeria, Nsukka

Simon, M.J., Langergren, E.S. and Snyder, K.A., Concrete mixture optimisation using statistical mixture design methods, In proceeding of the PCI/FHWA International symposium on high performance concrete New Orleans, 1997, pg 230-244.

Scheffe, H., (1958) Experiments with mixtures. *Royal Statistical Society Journal*, Series B. 20, 344-360.