Total Perfect Domination In Interval-Valued Fuzzy Graphs

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Abstract

A perfect dominating set D_p of an interval-valued fuzzy graph $G = (A, B)$ of $G^* = (V, E)$ is called a total perfect dominating set of G if every vertex of G is dominates to at least one vertex of D_p and denoted by D_{tp}. In This paper, the new kind of parameter total perfect domination in interval-valued fuzzy graphs is defined and studied. We research and concludes Some bounds on total perfect domination number $\gamma_{tp}(G)$ and upper total perfect domination number $\Gamma_{tp}(G)$ for several classes in interval-valued fuzzy graphs such as complete, star, bipartite, wheel, cycle and complete bipartite etc. Finally we introduce the properties between the concept of total perfect domination and other concepts like perfect domination, total domination and connected in interval-valued fuzzy graphs.

Keywords:- Interval-valued fuzzy graph, total dominating set, perfect dominating set , total Perfect dominating set, total perfect domination number.

2010 Mathematics Subject Classification: (03E72), (05C69), (05C72).

1 Introduction

After initial studies presented in (1975) by Zadeh and Rosenfeld [21, 16] about the fuzzy graph and the interval-valued fuzzy graph. Since then, graph theory evolved in many different areas, the most important of which was the subject of control and domination.
The concept of domination in fuzzy graphs was investigated by A. SomaSundaram and S. SomaSundaram [19] further A. SomaSundaram presented the concepts of independent domination, total domination of fuzzy graphs [20].
The concept of total domination in fuzzy graphs using strong arcs was introduced by Manjusha and Sunitha in (2015) [9].
Domination of an interval-valued fuzzy graph has wide applications in real-life. The concepts of domination in interval valued fuzzy graphs was investigated by Pradip Debnath [5], he introduced some results about the concept of total domination in interval valued fuzzy graphs. Sarala and Kavitha [17, 18] were published some papers in the concepts of strong (weak) domination in interval-valued fuzzy graphs, and complete and complementary domination number in interval-valued fuzzy graphs. Also there are other researchers who studied some properties and operations on the interval-valued fuzzy graphs, we refer to [1, 13, 7]. Philip and etc al [10, 11, 12] between (2017-2019) discussed the notions of interval-valued fuzzy bridges and interval-valued fuzzy cutnodes, different kinds of arcs in interval valued fuzzy graphs and some matrices associated with interval-valued fuzzy graph.
In (2013) Revathi etc al[14] introduced the notion of perfect domination in fuzzy graphs.
In (2021) we discussed the concept of perfect domination in interval-valued fuzzy graphs [2].
In this article we study and discuss the concept of total perfect domination number in an interval-valued fuzzy graphs and we introduce a new graph theoretic parameter known as total perfect domination number in interval-valued fuzzy graph. We give some important results and some examples.

2 Preliminaries

In this section, we collect some basic definitions related to interval-valued fuzzy graphs and domination in interval-valued fuzzy graphs.

Definition 2.1 *[1]* An $IVFG$ of the graph $G^* = (V, E)$ is a pair $G = (A, B)$ where, $A = [\mu_1, \mu_2]$ is an interval-valued fuzzy set on V and $B = [\rho_1, \rho_2]$ is an interval-valued fuzzy relation on V, such that

$$\rho_1(x, y) \leq \mu_1(x) \wedge \mu_1(y)$$

and

$$\rho_2(x, y) \leq \mu_2(x) \wedge \mu_2(y)$$

for all $(x, y) \in E$.

Definition 2.2 *[1]* Let $G = (A, B)$ be an $IVFG$. Then the order p and size q are defined to be $p = \sum_{v_i \in V} \frac{1 + \mu_2(v_i) - \mu_1(v_i)}{2}$ and $q = \sum_{(v_i,v_j) \in E} \frac{1 + \rho_2(v_i, v_j) - \rho_1(v_i, v_j)}{2}$.

Definition 2.3 *[1, 13]* Let $G = (A, B)$ of $G^* = (V, E)$ where $A = [\mu_1, \mu_2]$ and $B = [\rho_1, \rho_2]$. Then an $IVFG$, G is called complete $IVFG$ if $\rho_1(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\rho_2(x, y) = \mu_2(x) \wedge \mu_2(y)$, for all $x, y \in V$ and is denoted by K_μ. The complement of an $IVFG$, G is $IVFG$, $\overline{G} = (\overline{A}, \overline{B})$ where

$\overline{\mu_1} = \mu_1$; $\overline{\mu_2} = \mu_2$ for all vertices, also

$\overline{\rho_1}(x, y) = \mu_1(x) \wedge \mu_1(y) - \rho_1(x, y)$ and $\overline{\rho_2}(x, y) = \mu_2(x) \wedge \mu_2(y) - \rho_2(x, y)$ for all $x, y \in E$.

Definition 2.4 *[17]* An $IVFG$, G is called bipartite if the vertex set V of G can be partitioned into two non empty sub sets V_1 and V_2 such that $V_1 \cap V_2 = \phi$.

A bipartite $IVFG$, G is called complete bipartite $IVFG$ if $\rho_1(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\rho_2(v_i, v_j) = min\{\mu_2(v_i), \mu_2(v_j)\}$ for all $v_i \in V_1$ and $v_j \in V_2$. Its denoted by $K_{m,n}$, where $|V_1| = m, |V_2| = n$.

Definition 2.5 *[5]* Let $G = (A, B)$ be $IVFG$ and let $S \in V(G)$, then a vertex sub set S of G is said to be independent set if $\rho_1(u, v) < \mu_1(u) \wedge \mu_1(v)$, and $\rho_2(u, v) < \mu_2(u) \wedge \mu_2(v)$ or $\rho_1(u, v) = 0, \rho_2(u, v) = 0$ for all $u, v \in S$.

Definition 2.6 *[5]* Let $G = (A, B)$ be an $IVFG$ and let $u, v \in V(G)$. Then we say that u dominates v or v dominates u if (uv) is a effective edge, i.e $\rho_1(uv) = min(\mu_1(u), \mu_1(v))$ and $\rho_2(uv) = min(\mu_2(u), \mu_2(v))$. A vertex sub set $(D \subseteq V)$ of $V(G)$ is called dominating set in $IVFG$ G, if for every $v \in V - D$ there exists $u \in D$, such that (uv) is effective edge. A dominating set D of an $IVFG$ G, is called minimal dominating set if $D - \{u\}$ is not dominating set for every $u \in D$. A minimal dominating set D, with $|D| = \gamma(G)$ is denoted by $\gamma - set$.

Definition 2.7 *[5]* A dominating set D of an $IVFG$ G is said to be a total dominating set, if every vertex in V is dominated by a vertex in D. The minimum fuzzy cardinality of a total dominating set is called the total domination number of an $IVFG$ G and is denoted by $\gamma_t(G)$ or γ_t.

Definition 2.8 *[17]* Given a fuzzy graph G, choose $u \in V(G)$ and put $S = u$, for every u we have $N(S) = V - S$ denoted by $\acute{S}$ is the complete dominating set of an $IVFG$. The minimum cardinality of a complete dominating set of interval-valued fuzzy is called the complete domination number of G.

Definition 2.9 *[17]* Let $G = (A, B)$, be a Complementary $IVFG \quad G$ on V and $u, v \in V$, We say u dominates v if $\overline{\rho_1}(u, v) = min\{\mu_1(u), \mu_1(v)\}$ and $\overline{\rho_2}(u, v) = min\{\mu_2(u), \mu_2(v)\}$. A subset S of V is called a dominating set in $\overline{G}$ if for every $v \notin S$, there exists $u \in S$ such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(\overline{G})$.

Definition 2.10 *[5]* Let $G = (A, B)$ be $IVFG$ and let $S \in V(G)$, then a vertex sub set S of G is said to be independent set if $\rho_1(u, v) < \mu_1(u) \wedge \mu_1(v)$, and $\rho_2(u, v) < \mu_2(u) \wedge \mu_2(v)$ or $\rho_1(u, v) = 0, \rho_2(u, v) = 0$ for all $u, v \in S$.

Definition 2.11 *[2]* A dominating set D of an $IVFG$ G is called the perfect dominating set of G if for each vertex $v \in D$, v is dominates by exactly one vertex of D and is denoted by D_p. A perfect dominating set D_p of an $IVFG$ G is called minimal perfect dominating set, if for $u \in D_p$, $D_p - \{u\}$ is not perfect dominating set of G. The minimum fuzzy cardinality among all minimal perfect dominating sets of an $IVFG$, G is called the perfect domination number of G and denoted by $\gamma_p(G)$.

3 Total Perfect Domination in Interval-Valued Fuzzy Graphs

Definition 3.12 A perfect dominating set D_p of an interval-valued fuzzy graph $G = (A, B)$ of $G^* = (V, E)$ is said to be a total perfect dominating set of G if every vertex of G is dominates to at least one vertex of D_p and it is denoted by D_{tp}.

Definition 3.13 A total perfect dominating set D_{tp} of an interval-valued fuzzy graph G is called a minimal total perfect dominating set of G if for every $u \in D_{tp}$, $D_{tp} - \{u\}$ is not a total perfect dominating set of G.

Definition 3.14 The minimum fuzzy cardinality of all total perfect dominating sets of an interval-valued fuzzy graph G is called the total perfect domination number of G and is denoted by $\gamma_{tp}(G)$.

Definition 3.15 The maximum fuzzy cardinality of all minimal total perfect dominating sets of an interval-valued fuzzy graph G is called the upper total perfect domination number of G and it is denoted by $\Gamma_{tp}(G)$.

Remark 3.16 A total perfect dominating set of an interval-valued fuzzy graph G, with smallest fuzzy cardinality, i.e $|D_{tp}| = \gamma_{tp}(G)$ is called the minimum total perfect dominating set and denoted by $\gamma_{tp} - set$.

Remark 3.17 A total perfect dominating set of an interval-valued fuzzy graph G, with largest fuzzy cardinality, i.e $|D_{tp}| = \Gamma_{tp}(G)$ is called the maximum total perfect dominating set and denoted by $\Gamma_{tp} - set$.

Example 3.18 For the interval-valued fuzzy graph G given in the following Fig, such that all edges of G are effective.

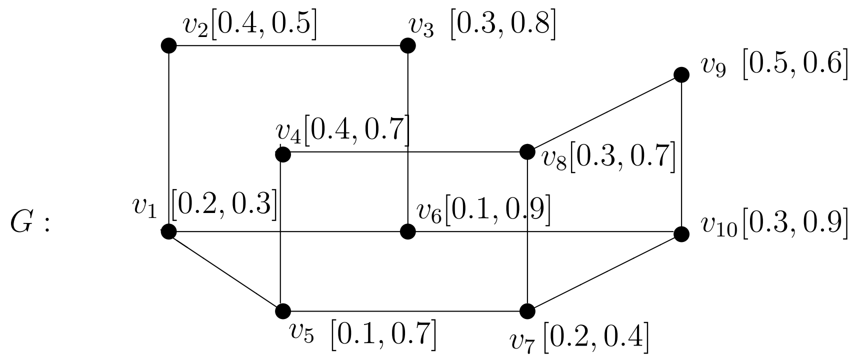

Fig 1

We see that, $S_1 = \{v_1, v_2, v_8, v_9\}$, $S_2 = \{v_2, v_3, v_7, v_8\}$, $S_3 = \{v_3, v_4, v_6, v_8\}$ and $S_4 = \{v_1, v_5, v_6, v_7, v_{10}\}$ are minimal total perfect dominating sets of G. Then $\gamma_{tp}(G) = min\{|S_1|, |S_2|, |S_3|, |S_4|\} = min\{2.35, 2.6, 3, 3.65\} = 2.35$.
And $\Gamma_{tp}(G) = max\{|S_1|, |S_2|, |S_3|, |S_4|\} = max\{2.35, 2.6, 3, 3.65\} = 3.65$. So that $\gamma_{tp} - set = |S_1|$ and $\Gamma_{tp} - set = S_4$.

Theorem 3.19 Every total perfect dominating set of an interval-valued fuzzy graph G is perfect dominating set Of G, but The converse need not is true.

proof 3.20 Let G be any interval-valued fuzzy graph without isolated vertices and D_{tp} be a total perfect dominating set of G. Then by the Definition of D_{tp}, a dominating set D_{tp} of an interval-valued fuzzy graph G is total perfect dominating set of G if every vertex $u \in V - D_{tp}$ is dominated by exactly one vertex in D_{tp} and for vertex of G is dominates to at least one vertex of D_{tp}. Since $u \in V - D_{tp}$, hence there $v \notin D_{tp}$ and u dominated by exactly one vertex in D_{tp}, which is a perfect dominating set of G. But the converse need not is true.

We will shows the converse of the above Theorem in the following Example.

Example 3.21 Consider the interval-valued fuzzy graph G given in the following Fig, such that every edge of G is strong edge.

G:

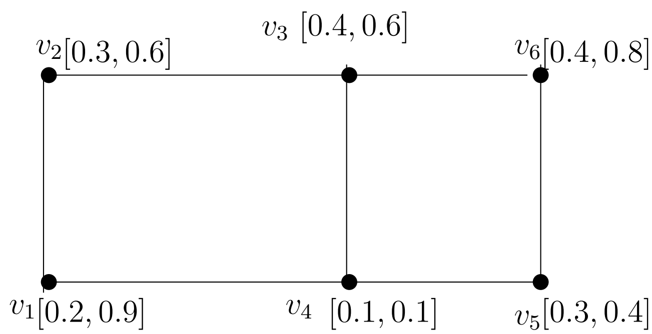

Fig 2

From the above Figure we have $D_1 = \{v_3, v_4\}$ is a total perfect dominating set of an interval-valued fuzzy graph G and also perfect dominating set of G. Whilst, $D_2 = \{v_2, v_5\}$ is perfect dominating set of an interval-valued fuzzy graph G, but it is not total perfect dominating set of G. Hence the converse of above theorem need not is true.

Corollary 3.22 Let $G = (A, B)$ be any interval-valued fuzzy graph without isolated vertices.

$$\gamma_p(G) \leq \gamma_{tp}(G)$$

.

proof 3.23 Let G be any interval-valued fuzzy graph without isolated vertices. Then by the above Theorem every total perfect dominating set of an interval-valued fuzzy graph is perfect dominating set of G. Hence clear that

$$\gamma_p(G) \leq \gamma_{tp}(G)$$

.

Theorem 3.24 Every total perfect dominating set of an interval-valued fuzzy graph G is total dominating set Of G. But the converse need not is true.

proof 3.25 Let G be any interval-valued fuzzy graph has no isolated vertex. Suppose that D_{tp} is total perfect dominating set of an interval-valued fuzzy graph. Hence clear that by the Definition of D_{tp} if every vertex of G is dominates to at least one vertex of D_{tp} which it is total dominating set of G. But The converse need not is true.

In the following Example, we shown that the two above Theorems are true.

Example 3.26 For the interval-valued fuzzy graph G in Fig (3), such that every edge of G is strong edge.

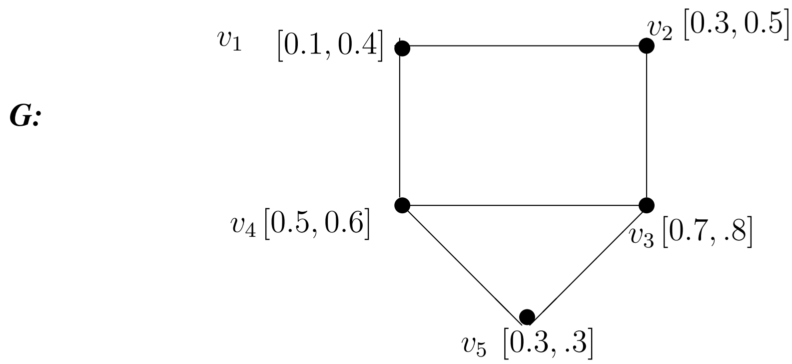

Fig 3

From the above Fig, we see that, a vertex subset of an interval-valued fuzzy graph G, $D_1 = \{v_1, v_4\}$ is a total perfect dominating set of an interval-valued fuzzy graph G and total dominating set of G. So that we have $D_2 = \{v_3, v_4\}$ is total dominating set of an interval-valued fuzzy graph G, but D_2 need not is total perfect dominating set of G. Therefor, i.e the converse of above theorem need not is true.

Corollary 3.27 Let G be any interval-valued fuzzy graph has no isolated vertex.

$$\gamma_t(G) \leq \gamma_{tp}(G)$$

.

proof 3.28 Let G be any interval-valued fuzzy graph has no isolated vertex. Then by the above Theorem every total perfect dominating set of an interval-valued fuzzy graph is total dominating set of G. Hence clear that

$$\gamma_t(G) \leq \gamma_{tp}(G)$$

.

Theorem 3.29 Every total perfect dominating set of an interval-valued fuzzy graph G need not be a connected perfect dominating set Of G.

proof 3.30 Let G be any connected interval-valued fuzzy graph and let D_{tp} be a total perfect dominating set of G. We know that, by definition of total perfect dominating set, if for each vertex $u \notin D_{tp}$, u dominated by exactly one vertex of D_{tp} and also each vertex of G is dominates to at least one vertex of D_{tp}. It is clear that, the induced sub graph ¡D_{tp}¿ has two-state: either connected perfect dominating set or not. Therefore, D_{tp} need not be connected perfect dominating set of an interval-valued fuzzy graph G.

Example 3.31 For the interval-valued fuzzy graph G given in the Fig (4), where every edge of G is strong edge.

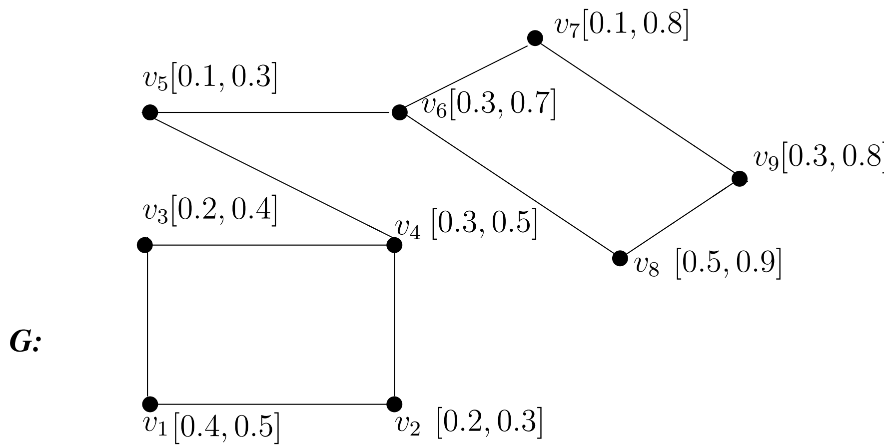

Fig 4

From the above Fig, we see that a vertex subset $D_{tp} = \{v_3, v_4, v_8, v_9\}$ is total perfect dominating set of an interval-valued fuzzy graph G, but it is not connected perfect dominating set of G.

Remark 3.32 Let $G = (A, B)$ be any interval-valued fuzzy graph has no isolated vertex. Then

$$\gamma(G) \leq \gamma_t(G) \leq \gamma_{tp}(G)$$

and

$$\gamma(G) \leq \gamma_p(G) \leq \gamma_{tp}(G)$$

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. But $\gamma_t(G) \neq \gamma_p(G)$, i.e every total dominating set of an interval-valued fuzzy graph G need not is perfect dominating set of G and every perfect dominating set of an interval-valued fuzzy graph G need not is total dominating set of G.

In the following Theorem, we introduce some properties related to this the total perfect dominating set of an interval-valued fuzzy graph G with perfect dominating set and total dominating set in interval-valued fuzzy graph G.

Theorem 3.33 Let G be interval-valued fuzzy graph with $\gamma_{tp} - set$ of G. Then a total dominating set of an interval-valued fuzzy graph is perfect dominating set of G if only if $\gamma_{tp} - set$ is total perfect dominating set of G.

proof 3.34 Let G be any interval-valued fuzzy graph with $\gamma_{tp} - set$ of G. suppose that S_1 is minimal total dominating set of an interval-valued fuzzy graph G and S_2 is minimal perfect dominating set of an interval-valued fuzzy graph G. Then we have two cases:
Case1: If $S_1 \neq S_2$, which is not total perfect dominating set of G, it is contradicting to the Theorem.
Case2: If $S_1 = S_2$, which $\gamma_{tp} - set$ both total dominating set and perfect dominating set of G. Therefore $\gamma_{tp} - set$ is total perfect dominating set of G.
Conversely: Suppose that $\gamma_{tp} - set$ be a total perfect dominating set of an interval-valued fuzzy graph G. By above theorems this the prove is obvious.

Example 3.35 Consider the interval-valued fuzzy graph G, where all edges of G are strong.

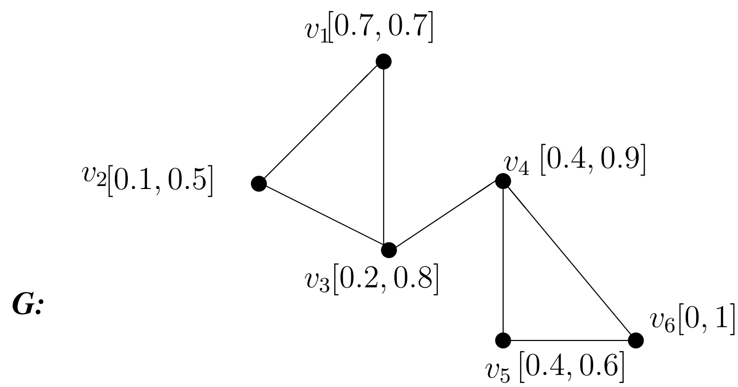

Fig 5

From the abov Fig, we see that $D_1 = \{v_1, v_2, v_5, v_6\}$ is total dominating set of an interval-valued fuzzy graph G, but is not perfect dominating set of G. And $D_2 = \{(v_1, v_5)\}$ is perfect dominating set of an interval-valued fuzzy graph G, but is not total dominating set of G. Also we have $D_3 = \{v_3, v_4\}$ is total perfect dominating set of an interval-valued fuzzy graph G, so that it is both total dominating set and perfect dominating set of G.

Corollary 3.36 Let G be total interval-valued fuzzy graph with $\gamma_{tp} - set$ of G. Then a perfect dominating set of an interval-valued fuzzy graph is total dominating set of G if only if $\gamma_{tp} - set$ is total perfect dominating set of G.

proof 3.37 By above theorem, the proof it is obvious.

Remark 3.38 For any interval-valued fuzzy graph, G if $V(G)$ is an independent. Then

$$\gamma_{tp}(G) = 0$$

and

$$\Gamma_{tp}(G) = 0$$

.

Theorem 3.39 Let G be interval-valued fuzzy graph without isolated vertices, $n \geq 5$ and D_{tp} be a $\gamma_{tp} - set$ of G. Then $V - D_{tp}$ need not be total perfect dominating set of G.

proof 3.40 Let G be interval-valued fuzzy graph without isolated vertex, $n \geq 5$ and D_{tp} be a minimum total perfect dominating set of G. Suppose that $V - D_{tp}$ is a total perfect dominating set of G. Then by the definition of Perfect dominating set, for each vertex $u \notin V - D_{tp}$, u is dominated by exactly one vertex of $V - D_{tp}$ and by the definition of total Perfect dominating set, the perfect dominating set is total perfect dominating set if every vertex of G is dominates to at least one vertex of $V - D_{tp}$. Hence there is vertex $v \in V - D_{cp}$, such that v does not adjacent to any vertex in D_{tp} or v dominates to at least one vertex in D_{tp}. Therefor D_{tp} is not minimum total perfect dominating set of G, which is a contradiction. Thus $V - D_p$ need not total perfect dominating set of G.

Example 3.41 For the interval-valued fuzzy graph G given in the Fig (6), where every edge of G is strong edge.

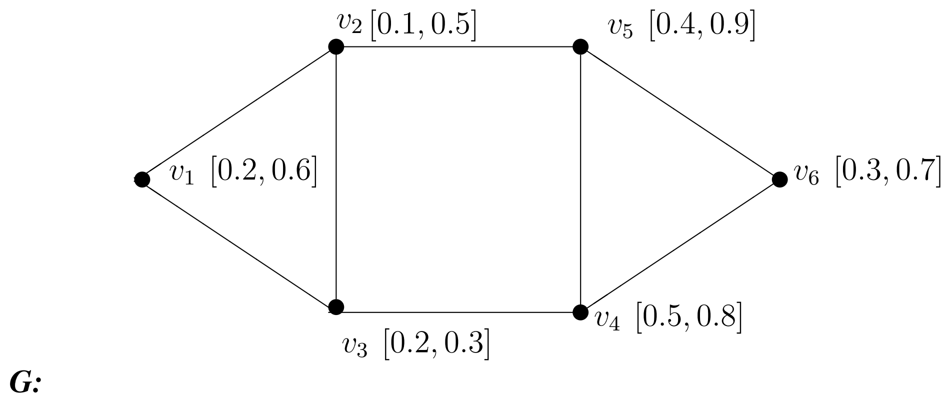

Fig 6

From the above Fig, we see that a vertex subset $D_{tp} = \{v_3, v_4, \}$ is minimum total perfect dominating set of an interval-valued fuzzy graph G. Then $V - D_{tp} = \{v_1, v_2, v_5, v_6\}$ need not be total perfect dominating set of G.

Theorem 3.42 For any interval-valued fuzzy graph G has no isolated vertex and with D_{tp} be a total perfect dominating set of G. If each $u \in D_{tp}$ is dominated by exactly one vertex of $V - D_{tp}$. Then $V - D_{tp}$ is perfect dominating set of G.

proof 3.43 Let G be any interval-valued fuzzy graph has no isolated vertex, and D_{tp} be a total perfect dominating set of G. Suppose that u be any a vertex in D_{tp}. Then if each $u \in D_{tp}$ is dominated by exactly one vertex of $V - D_{tp}$, since D_{tp} be a total perfect dominating set of G, i.e for $v \in V - D_{tp}$ is dominated by only one vertex of D_{tp}. Thus the number of vertices in D_{tp} equal the number of vertices in $v - D_{tp}$. Therefor $V - D_{tp}$ is perfect dominating set of G.

Example 3.44 consider the interval-valued fuzzy graph G given in Fig 7, such that all edges in G are strong.

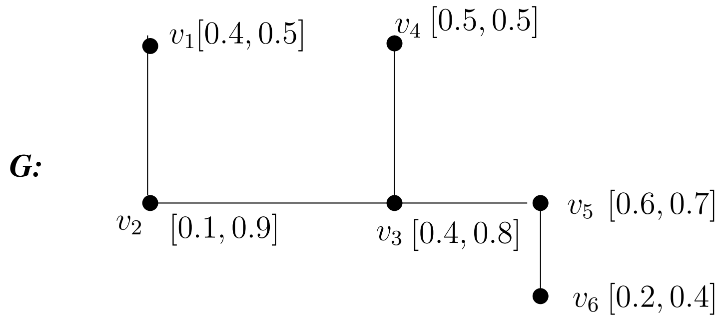

Fig 7

From the above Fig, we see that, $D_{tp} = \{v_2, v_3, v_6\}$ is total perfect dominating set of G. Then $V - D_{tp} = \{v_1, v_4, v_6\}$ is perfect dominating set of G.

Remark 3.45 Every connected perfect dominating set of an interval-valued fuzzy graph G need not be total perfect dominating set Of G.

Example on that the complete and the wheel. And below the show it.

Remark 3.46 for any complete interval-valued fuzzy graph G. Then total dominating set of G is exists.

Theorem 3.47 Total perfect dominating set is not exists for any complete interval-valued fuzzy graph $G = k_{\mu_A}$.

proof 3.48 Let G be any complete interval-valued fuzzy graph. Then all edges in K_{μ_A} are strong edges and each vertex in k_{μ_A} dominates to all the other vertices in μ_A. Hence a perfect dominating set is exists and contain exactly one vertex say v in k_{μ_A}, where v has minimum membership value in k_{μ_A}. Since it has only one vertex and does not achieve the definition of total perfect dominating set of an interval-valued fuzzy graph G. Thus total perfect dominating set of an interval-valued fuzzy graph G is not exists for any complete interval-valued fuzzy graph $G = k_{\mu_A}$.

Corollary 3.49 Let G be any complete interval-valued fuzzy graph and let $\overline{G}$ be the complementary of G. Then total perfect dominating set in $\overline{G}$ is not exists.

Corollary 3.50 For any complete interval-valued fuzzy graph G.

$$\gamma_{tp}(k_\mu) = \gamma_{tp}(\overline{k_\mu}) = 0.$$

In the following we give Γ_{tp} and γ_{tp} for The complete bipartite interval-valued fuzzy graph G.

Remark 3.51 Every connected perfect dominating set of G is total perfect dominating set of G for any complete bipartite interval-valued fuzzy graph G and the converse need is true.

Theorem 3.52 For any complete bipartite interval-valued fuzzy graph, $G = k_{m,n}$. Then

$$\Gamma_{tp}(G) = max\{|x| : x \in V_1\} + max\{|y| : y \in V_2\}.$$

And

$$\gamma_{tp}(G) = min\{|x| : x \in V_1\} + min\{|y| : y \in V_2\}.$$

proof 3.53 Let $G = k_{m,n}$ be complete bipartite interval-valued fuzzy graph, where m, n are the membership value in vertices V_1 and V_2 respectively. Then all edges in G are effective and each vertex in V_1 dominates to all vertices in V_2 and each vertex in V_2 dominates to all vertices in V_1. Hence perfect dominating set of G contain exactly two vertices x, y, where $x \in V_1$ and $y \in V_2$, which is satisfy the definition of total perfect dominating set of G. Thus total perfect dominating set of G is exists and $D_{tp} = \{x, y\}$. Hence x has the maximum membership value in V_1 and y has the maximum membership value of V_2. Similarly x has the minimum membership value in V_1 and y has the minimum membership value of V_2 Therefor,

$$\Gamma_{tp}(G) = max\{|x| : x \in V_1\} + max\{|y| : y \in V_2\}$$

and

$$\gamma_{tp}(G) = min\{|x| : x \in V_1\} + min\{|y| : y \in V_2\}.$$

Example 3.54 Consider the interval-valued fuzzy graph G, shown in Fig 8, such that all edges of G are effective.

G:

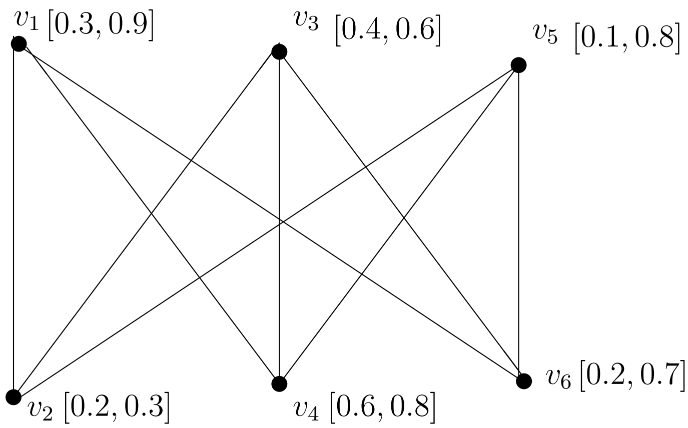

Fig 8

From the above Fig , we see that a vertex subset $\{v_1, v_2\}$, $\{v_1, v_4\}$, $\{v_1, v_6\}$, $\{v_2, v_3\}$, $\{v_3, v_4\}$, $\{v_3, v_6\}$, $\{v_2, v_5\}$, $\{v_4, v_5\}$ and $\{v_5, v_6\}$ are minimal total perfect dominating sets of an interval-valued fuzzy graph G. Then a minimum total perfect dominating set of $G = \{v_2, v_3\}$. Hence $\gamma_{tp}(G) = 1.15$ And maximum total perfect dominating set of $G = \{v_1, v_6\}$. Hence $\Gamma_{tp}(G) = 1.55$.

Corollary 3.55 Let $G = k_{n,m}$ be a complete bipartite interval-valued fuzzy graph. Then

$$\gamma_p(G) = \gamma_{tp}(G) = \gamma_{cp}(G).$$

and

$$\Gamma_p(G) = \Gamma_{tp}(G) = \Gamma_{cp}(G).$$

Theorem 3.56 For any complete bipartite interval-valued fuzzy graph $G = k_{n,m}$ and D_{tp} be a minimal total perfect dominating set of $k_{n,m}$. Then $V - D_{tp}$ is a total dominating set of $k_{n,m}$.

proof 3.57 Let $G = k_{n,m}$ be complete bipartite interval-valued fuzzy graph, where m, n are the membership value in vertices V_1 and V_2 respectively. Then all edges in G are effective and each vertex in V_1 dominates to all vertices in V_2 and each vertex in V_2 dominates to all vertices in V_1. Hence a total perfect dominating set of G contains exactly two vertices x, y where $x \in V_1$ and $y \in V_2$. Hence $V - D_{tp} = V_1 - \{x\} + V_2 - \{y\}$ is a total dominating set of $G = k_{n,m}$.

Corollary 3.58 Let $G = k_{n,m}$ be a complete bipartite interval-valued fuzzy graph. Then

$$\gamma_{tp}(G) = \gamma_p(\overline{G}).$$

Remark 3.59 For any complete bipartite interval-valued fuzzy graph G, with order p. Then $\gamma_p(G) \leq \frac{p}{2}$.

In the following example we show that If (G) is a complete bipartite, the results above is true.

Example 3.60 Consider the interval-valued fuzzy graph G, shown in the following Fig, such that all edges of G are strong.

G:

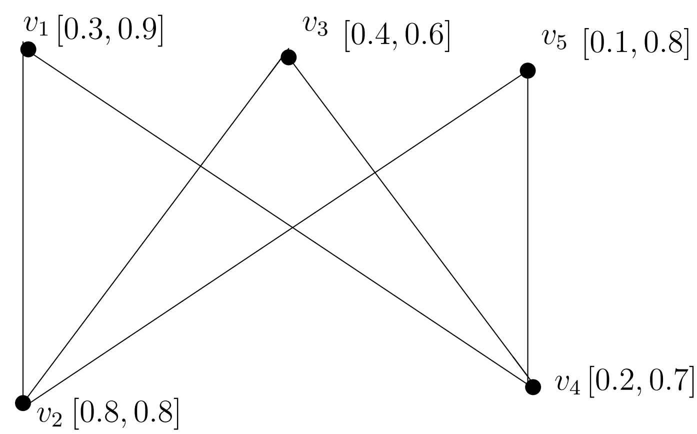

Fig 9

From the above Fig, we see that, $D_{tp} = \{v_2, v_3\}$ is total perfect dominating set of an interval-valued fuzzy graph G. Hence $\gamma_{tp}(G) = 1.1$, and $p = 3.5$. Therefore $\frac{p}{2} = \frac{3.5}{2} = 1.75$. Thus $\gamma_{tp}(G) \leq \frac{p}{2}$ is true.

In the following we give $\gamma_{tp}(G)$ and $\Gamma_{tp}(G)$ for The star interval-valued fuzzy graph G.

Theorem 3.61 Let G be a strong star interval-valued fuzzy graph. Then

$$\gamma_{tp}(G) = |u| + min|v_i|,$$

and

$$\Gamma_{tp}(G) = |u| + max|v_i|,$$

such that u is a root vertex and $v_i = V - \{u\}$ for $i = \{1, 2, ..., n - 1\}$.

proof 3.62 Let G be a strong star interval-valued fuzzy graph, then the vertex set of G are $\{u, v_i\}$, where u the root vertex of G and $v_i = \{v_1, v_2, ..., v_{n-1}\}$.Hence u dominates to all vertices of v_i, for $i = \{1, 2, ..., n-1\}$. Here suppose v_1 is the minimum of v_i or v_1 is the minimum of $V - \{u\}$ and suppose v_2 is the maximum of v_i or v_2 is the maximum of $V - \{u\}$. Hence minimum total perfect dominating set of G contains two vertices u and v_1 and maximum total perfect dominating set of G contains two vertices u and v_2. Therefore, the minimum total perfect domination number

$$\gamma_{tp}(G) = |u| + min|v_i| = |u| + |v_1|$$

and the maximum total perfect domination number

$$\Gamma_{tp}(G) = |u| + max|v_i| = |u| + |v_2| : \ u$$

is a root vertex and $v_i = V - \{u\}$ for $i = \{1, 2, ..., n-1\}$.

Example 3.63 For the interval-valued fuzzy graph G, shown in the following Fig, such that all edges of G are strong.

G:

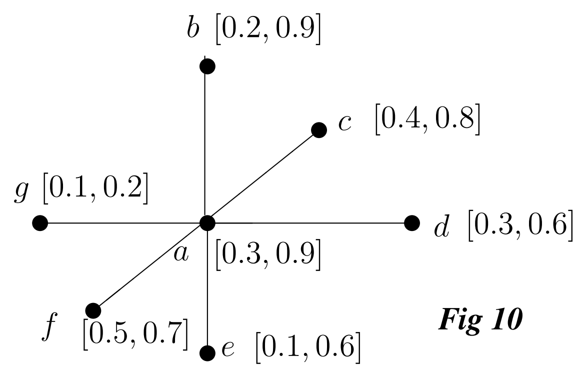

Fig 10

From the above Fig, we have $V = \{a, b, c, d, e, f, g\}$, such that $\{a\}$ is a root vertex and $\{g\}$ is the minimum of $V - \{a\}$. Then total perfect dominating set of a star interval-valued fuzzy graph G contains two vertices a and g. Thus total perfect domination number $= \gamma_{tp}(G) = |a| + |g| = 0.6 + 0.55 = 1.15$. Also we have $\{b\}$ is the maximum of $V - \{a\}$. Then upper total perfect dominating set of a star interval-valued fuzzy graph G contains two vertices a and b. Thus upper total perfect domination number $= \Gamma_{tp}(G) = |a| + |b| = 0.6 + 0.85 = 1.45$.

Corollary 3.64 For any strong star interval-valued fuzzy graph G,

$$\gamma_p(G) < \gamma_{tp}(G)$$

and

$$\Gamma_p(G) < \Gamma_{tp}(G).$$

proof 3.65 Let G be any strong star interval-valued fuzzy graph, then the vertex set of G are $\{u, v_i\}$, where u the root vertex of G and $v_i = \{v_1, v_2, ..., v_{n-1}\}$. Hence perfect dominating set of G contains one a vertex u such that u is a root vertex of a star interval-valued fuzzy graph G. Then perfect domination number $\gamma_{tp}(G) = |u| :\ u$ is a root vertex. Since the total perfect domination number $\gamma_{tp}(G) = |u| + min|v_i|$ and $\Gamma_{tp}(G) = |u| + max|v_i|$ by above Theorem. Therefore,

$$\gamma_p(G) < \gamma_{tp}(G)$$

and

$$\Gamma_p(G) < \Gamma_{tp}(G).$$

In the following we show γ_{tp} is not exists for The wheel interval-valued fuzzy graph G.

Theorem 3.66 For any strong wheel interval-valued fuzzy graph G. Total perfect dominating set of G is not exists.

proof 3.67 Let G be any strong wheel interval-valued fuzzy graph. Then the vertex set of $V(G)$ are $\{u, v_i\}$, $i = \{1, 2, ..., n-1\}$, where u the root vertex of G, and $v_i = \{v_1, v_2, ..., v_{n-1}\}$. Since a root vertex u dominates to v_i, for $i = \{1, 2, ..., n-1\}$. Hence perfect dominating set of G is exists and contains only the a root vertex u. Therefore the perfect domination number $\gamma_p(G) = |u|$, u is a root vertex. Since it has only one vertex and does not achieve the definition of total perfect dominating set of an interval-valued fuzzy graph G. So total perfect dominating set of an interval-valued fuzzy graph G is not exists for any strong wheel interval-valued fuzzy graph G.

Example 3.68 Consider the interval-valued fuzzy graph G, shown in the following Fig, such that all edges of G are strong.

G:

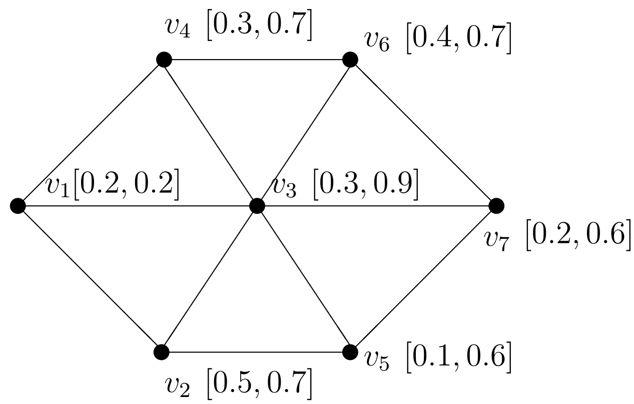

Fig 11

From the above Fig, we have $\{v_3\}$ is a root vertex of G, and $\{v_3\}$ anly is minimum perfect dominating set of wheel interval-valued fuzzy graph G. Then perfect dominating set of G is exists. But total perfect dominating set of G is not exists.

In the following we give γ_{tp} and for The cycle interval-valued fuzzy graph G.

Theorem 3.69 Let $G = C_n$ be any cycle interval-valued fuzzy graph, $n \geq 4$. Then a total perfect dominating set of C_n is exists and

$$\gamma_p(G) \leq \gamma_{tp}(G) \leq \gamma_{cp}(G)$$

, such that n the number of vertices of G.

proof 3.70 Let $G = C_n$ be a cycle interval-valued fuzzy graph, such that n the number of vertices of G. Then If $n = 4$ or $n = 5$. Hence a total perfect dominating set contains two vertices x and y of C_4 or contains three vertices of C_5. Thus $\gamma_p(G) = \gamma_{tp}(G) = \gamma_{cp}(G)$. If $n = 6$ or $n = 7$. Hence D_p contains two vertices or contains three vertices but $D_{tp} = D_{cp}$ and contains four vertices or contains fife vertices, thus $\gamma_p(G) < \gamma_{tp}(G) = \gamma_{cp}(G)$. If $n = 8$. Hence $D_p = D_{tp}$ and contains four vertices but D_{cp} contains six vertices. Thus $\gamma_p(G) = \gamma_{tp}(G) < \gamma_{cp}(G)$ ...etc. Therefore a total perfect dominating set is exists for any cycle interval-valued fuzzy graph $G = C_n$, where $n \geq 4$. Hence

$$\gamma_p(G) \leq \gamma_{tp}(G) \leq \gamma_{cp}(G)$$

.

In the following Example, we show that total perfect dominating set of an interval-valued fuzzy graph in above theorem is exists for any cycle of G, $n \geq 4$.

Example 3.71 For the interval-valued fuzzy graph $G = C_9$ in the following Fig, where all edges of G are effective.

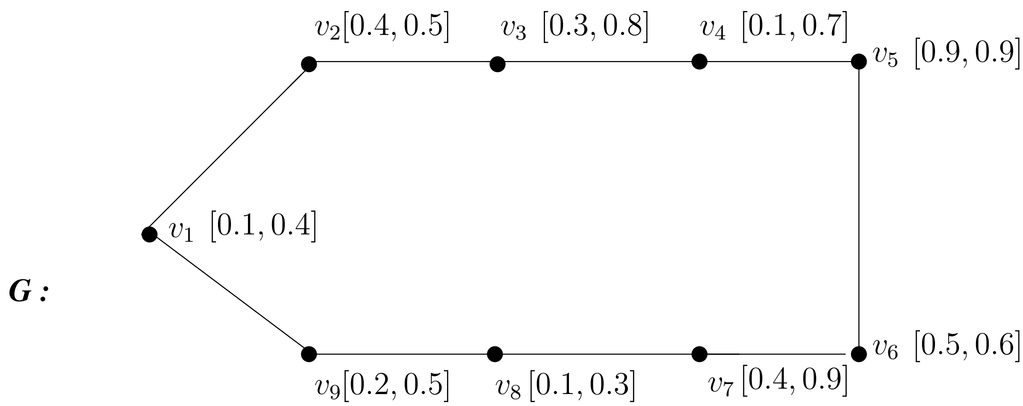

Fig 12

From the above Fig, we see that a vertex subset
$D_1 = \{v_2, v_5, v_8\}$ is minimum perfect dominating set of G. Then $\gamma_p(G) = \{0.55, 0.5, 0.6\} = 1.65$.
And $D_2 = \{v_1, v_2, v_5, v_6, v_9\}$ is minimum total perfect dominating set of G. Then $\gamma_{tp}(G) = \{0.65, 0.55, 0.5, 0.55, 0.65\} = 2.9$.
And $D_3 = \{v_1, v_2, v_5, v_6, v_7, v_8, v_9\}$ is minimum connected perfect dominating set of G. Then $\gamma_{cp}(G) = \{0.65, 0.55, 0.5, 0.55, 0.75, 0.6, 0.65\} = 4.25$. Therefore

$$\gamma_p(G) \leq \gamma_{tp}(G) \leq \gamma_{cp}(G)$$

.

Remark 3.72 For any strong cycle interval-valued fuzzy graph $G = C_n$, $n \geq 4$, such that n the number of vertices in G. Then total perfect dominating set of G, it has four cases:
(1) If $n = \{4, 8, 12, 16, ..., 4m\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n}{2}$.
(2) If $n = \{5, 9, 13, 17, ..., 4m + 1\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n+1}{2}$.
(3) If $n = \{6, 10, 14, 18, ..., 4m + 2\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n+2}{2}$.
(4) If $n = \{7, 11, 15, 19, ..., 4m + 3\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n+3}{2}$.

Example 3.73 For the interval-valued fuzzy graph G, shown in the following Fig, such that all edges of G are strong.

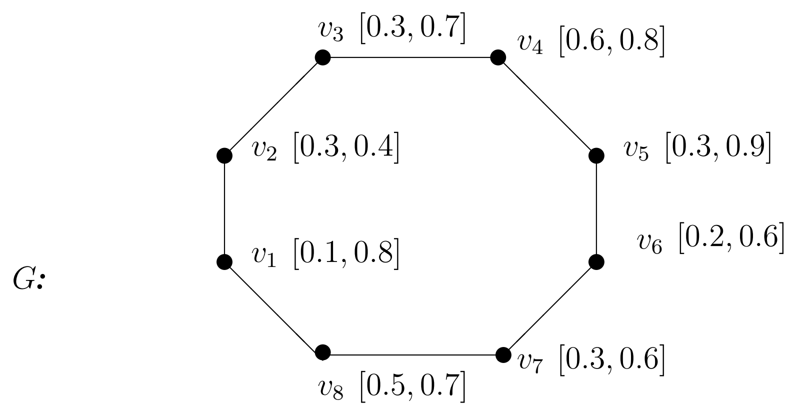

Fig 13

From the above Fig, we have $D_{tp} = \{v_2, v_4, v_7, v_8\}$ is a minimum total perfect dominating set of $G = C_8$. Since $n = 8$, by the above Remark if $n = \{4, 8, 12, 16, ..., 4m\}$ for $m = 1, 2, 3..., m \in N$. Hence $D_{tp} = \frac{n}{2} = \frac{8}{2} = 4$, such that n the number of vertices in G, i.e total perfect dominating set of $G = C_8$ has four vertices.

Theorem 3.74 For any cycle interval-valued fuzzy graph G, $n \geq 4$. Then total perfect domination number of G, it has two cases:

(1) If G is even, hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+k}{2}} |u_i|\right)$$

, such that $k = 0$ or $k = 2$.

(2) If G is odd, hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+k}{2}} |u_i|\right)$$

, such that $k = 1$ or $k = 3$.

proof 3.75 Let $G = C_n$ be a cycle interval-valued fuzzy graph, such that n the number of vertices in G. Then

(1) If G is even, by the above Remark $D_{tp} = \frac{n}{2}$, where $n = 4, 8, 12, ...4m$, $m \in N$ and $k = 0$

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or $D_{tp} = \frac{n+2}{2}$, where $n = 6, 10, 14, ..., 4m + 2$ and $k = 2$. Hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+k}{2}} |u_i|\right)$$

, such that $k = 0$ or $k = 2$.

(2) If G is odd, by the above Remark $D_{tp} = \frac{n+1}{2}$, where $n = 5, 9, 13, ...4m + 1$, $m \in N$ and $k = 1$ or $D_{tp} = \frac{n+3}{2}$, where $n = 7, 11, 15, ...4m + 3$, $m \in N$ and $k = 3$. Hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+k}{2}} |u_i|\right)$$

, such that $k = 1$ or $k = 3$.

Remark 3.76 For any strong path interval-valued fuzzy graph $G = P_n$, $n \geq 5$, such that n the number of vertices in G. Then total perfect dominating set of G, it has two cases:

Case1: If $n = \{5, 9, 13, ..., 4m + 1\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n+1}{2}$.

Case2: There are different solutions as follows: If $n = \{6, 7, 8\}$. Hence $D_{tp} = 4$ vertices.

(3) If $n = \{10, 11, 12\}$. Hence $D_{tp} = 6$ vertices.

(4) If $n = \{14, 15, 16\}$. Hence $D_{tp} = 8$. Thus ...etc.

Theorem 3.77 Let G be a path interval-valued fuzzy graph, $n \geq 5$. Then if G is odd and $n = 5, 9, 13, ..., 4m + 1$, $m \in N$. Hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+1}{2}} |u_i|\right)$$

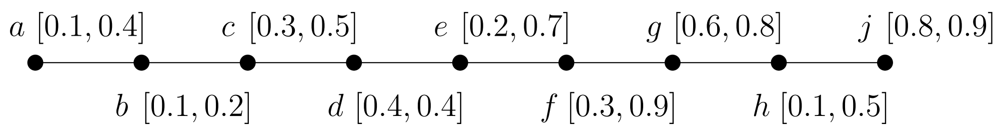
, where n the number of vertices in G.

proof 3.78 Let G be interval-valued fuzzy graph. Then a path P_n, has total perfect dominating set of G, $n \geq 5$. By the above Remark if $n = \{5, 9, 13, ..., 4m + 1\}$ for $m = 1, 2, 3...$, $m \in N$. Hence $D_{tp} = \frac{n+1}{2}$. Therefore

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{n+1}{2}} |u_i|\right)$$

, where n the number of vertices in G.

Example 3.79 Consider the interval-valued fuzzy graph G, given in the following Fig, such that all edges of G are strong.



G: ***Fig 14***

.

From the above Fig, we have $S_1 = \{b, c, f, g, h\}$, $S_2 = \{a, b, e, f, g, h\}$, $S_3 = \{b, c, d, g, h\}$ and $S_4 = \{b, c, d, e, h, j\}$ are minimal total perfect dominating sets of interval-valued fuzzy graph G. Since S_1 and S_3 are minimum total perfect dominating sets. Then total Perfect domination number of G

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{\frac{9+1}{2}} |u_i|\right)$$

. Hence

$$\gamma_{tp}(G) = min\left(\sum_{i=1}^{5} |S_1, S_3|\right) = \sum_{i=1}^{5} |S_3| = 2.95$$

.

Remark 3.80 Let G be a cycle or path interval-valued fuzzy graph, $n \geq 4$. Then every connected perfect dominating set of G is total perfect dominating set of G.

Theorem 3.81 Let $G = (A, B)$ of $G^* = (V, E)$ be any connected interval-valued fuzzy graph and $H = (A_1, B_1)$ of $H^* = (V_1, E_1)$ be any maximum spanning tree of an interval-valued fuzzy graph G. Then every total perfect dominating set of H is also a total perfect dominating set of G and $\gamma_{tp}(G) \leq \gamma_{tp}(H)$.

proof 3.82 Let D_{tp} be a total perfect dominating set of an interval-valued fuzzy graph H. Since H is maximum spanning tree of an interval-valued fuzzy graph G. Then we have $A =$

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A_1 and $V = V_1$. Hence the vertices in D_{tp} dominates to all the vertices in $V - D_{tp}$. Therefore D_{tp} is a total perfect dominating set of an interval-valued fuzzy graph$G$ and $\gamma_{tp}(G) \leq \gamma_{tp}(H)$.

Example 3.83 For the interval-valued fuzzy graph G and the interval-valued fuzzy graph H in Fig (15) and Fig (16), such that each edges of G and H are strong edges.

G:

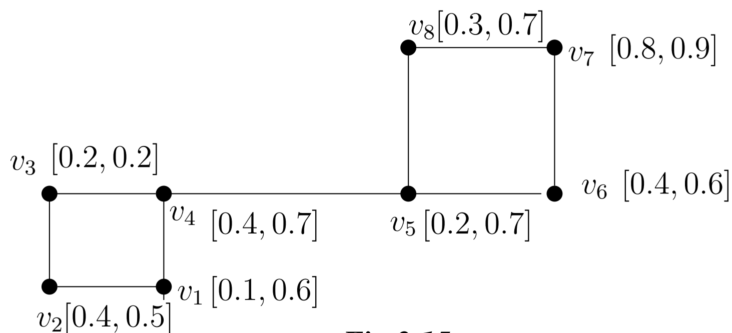

Fig 3.15

H:

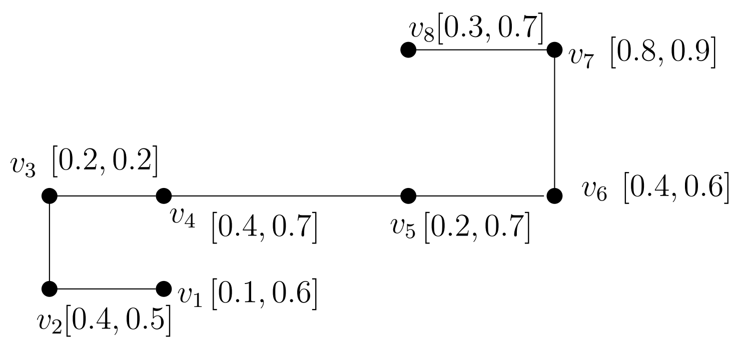

Fig 3.16

We see that, $D_{tp}(G) = \{v_2, v_3, v_6, v_7\}$ is a minimum total perfect dominating set of an interval-valued fuzzy graph G. Then a total perfect domination number of G, $\gamma_{tp}(G) = 2.2$. Also we see that, $D_{tp}(H) = \{v_2, v_3, v_6, v_7\}$ is a minimum total perfect dominating set of an interval-valued fuzzy graph H. Then a total perfect domination number of H, $\gamma_{tp}(H) = 2.2$. Hence $\gamma_{tp}(G) = \gamma_{tp}(H)$. Therefor $\gamma_{tp}(G) \leq \gamma_{tp}(H)$.

Theorem 3.84 Let G be any interval-valued fuzzy graph. Then $\gamma_{tp}(G) = p$ if and only if every vertex of G has a unique neighbor, where $p = |V|$.

proof 3.85 Let G be any interval-valued fuzzy graph and let $\gamma_{tp}(G) = p$. Suppose that x, y and z are vertices in G, such that x has two neighbors y and z. Then $x - \{z\}$ is a total perfect dominating set of G, so that $\gamma_{tp}(G) < p$, it is contradiction. Thus every vertex of G has a

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unique neighbor.

Conversely: Suppose that every vertex of an interval-valued fuzzy graph G has a unique neighbor. Hence total perfect dominating set of G is only V, where V be each the vertices of G. Therefor $\gamma_{tp}(G) = p$.

Example 3.86 For the interval-valued fuzzy graph G in the following Fig, where every vertex of G has a unique neighbor.

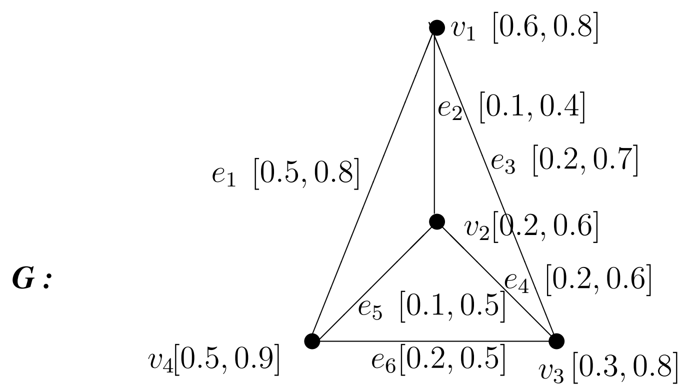

Fig 17

From the above Fig, we see that $N(v_1) = v_4, N(v_2) = v_3, N(v_3) = v_2$ and $N(v_4) = v_1$. So that $\gamma_{tp} = |v_1| + |v_2| + |v_3| + |v_4| = 0.6 + 0.7 + 0.75 + 0.7 = 2.75 = p$.

Corollary 3.87 For any interval-valued fuzzy graph G, if $\gamma_{tp}(G) = p$. Then the number of the vertices in G is even.

proof 3.88 Let G be interval-valued fuzzy graph. Suppose that G has odd number of vertices say $2n + 1$. By the above Theorem every vertex has a unique neighbor and since G no loops. Hence the number of the vertices in G is even.

Example 3.89 For the interval-valued fuzzy graph G given in the following Fig, we see that, $D_{tp} = \{a, b, c, d\} = V$. Then $\gamma_{tp}(G) = p$.

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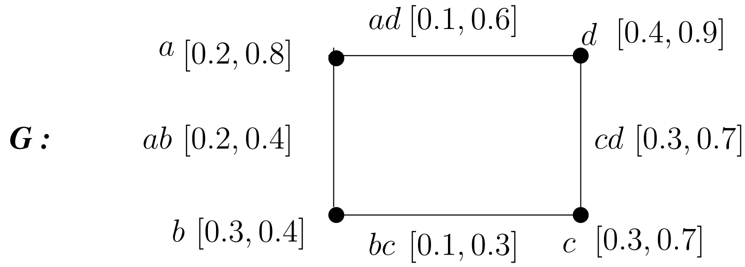

Fig 18

Remark 3.90 Let G and $\overline{G}$ be two interval-valued fuzzy graph, with effective edges. Then $\gamma_{tp}(G) + \overline{\gamma}_{tp}(G) < 2p$.

Example 3.91 consider the interval-valued fuzzy graph G and $\overline{G}$ shows in the Fig (19) and the Fig (20).

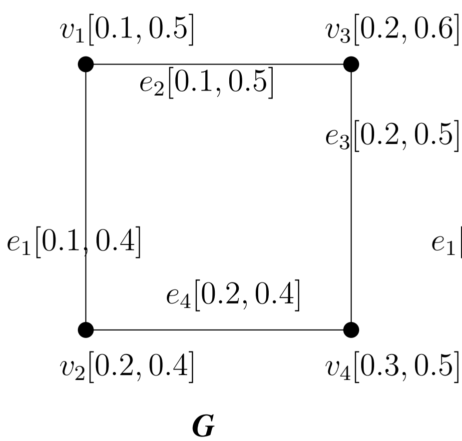

Fig 19

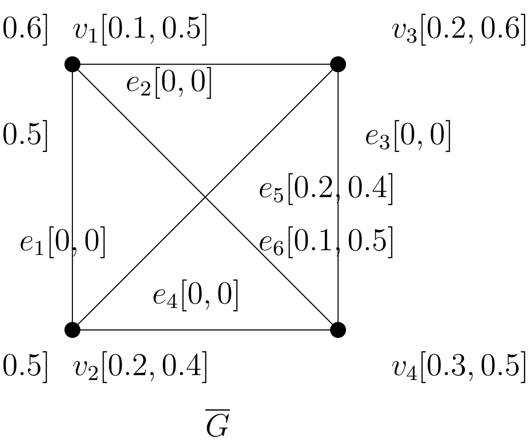

Fig 20

From the Fig (19), we see that a vertex subset $D_{tp} = \{v_2, v_4\}$ is minimal total perfect dominating set of G. Then $\gamma_{tp}(G) = 1.2$. Also from the Fig (20), we see that a vertex subset $\{v_1, v_4\}$ and $\{v_2, v_3\}$ are the two quarters in $\overline{G}$. Then $D_{tp} = \{v_1, v_2, v_3, v_4\}$ is total perfect dominating set of $\overline{G}$ and $\overline{\gamma}_{tp}(G) = p = 2.6$. So $1.2 + 2.6 < 2p = 5.2 \Rightarrow 3.8 < 5.2$. Therefore, $\gamma_{tp}(G) + \overline{\gamma}_{tp}(G) < 2p$.

Theorem 3.92 For any interval-valued fuzzy graph G, with at least one total perfect domi-

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nating set of G.

$$\gamma_p(G) \leq \frac{p + \gamma_{tp}(G)}{3}$$

.

proof 3.93 Let G be interval-valued fuzzy graph, with $\gamma_{tp} - set$. since $\gamma_p(G) \leq \gamma_{tp}(G)$. Then clear that $\gamma_p(G) \leq p + \gamma_{tp}(G)$. Hence

$$\gamma_p(G) \leq \frac{p + \gamma_{tp}(G)}{3}.$$

Remark 3.94 For any interval-valued fuzzy graph G. Then

(1) $\gamma_{tp}(G) \leq p - \Delta_N(G)$, it is trivial, or $\gamma_{tp}(G) \geq p - \Delta_N(G)$.

(2) $\gamma_{tp}(G) \leq p - \Delta_E(G)$, it is trivial, or $\gamma_{tp}(G) \geq p - \Delta_E(G)$.

Theorem 3.95 For any interval-valued fuzzy graph G, without isolated vertices.

$$\gamma_{tp}(G) \leq p - \Delta_N(G) + 1.$$

proof 3.96 Let G be any interval-valued fuzzy graph has no isolated vertex, with $\gamma_{tp} - set$ and let v be a vertex of G with $\Delta_N(v) = d_N(v)$. Then by above Remark, either $\gamma_{tp}(G) \leq p - \Delta_N$ or $\gamma_{tp}(G) \geq p - \Delta_N$. So, we have two cases:

Case1: If $\gamma_{tp} \leq p - \Delta_N(G)$. Then the proof is trivial.

Case2: If $\gamma_{tp}(G) \geq p - \Delta_N(G)$. Clear $\gamma_{tp}(G) - 1 \leq p - \Delta_N(G)$. Therefore

$$\gamma_{tp}(G) \leq p - \Delta_N(G) + 1.$$

Example 3.97 consider the interval-valued fuzzy graph G, shows in the Fig (21), where all arcs of G are strong.

G:

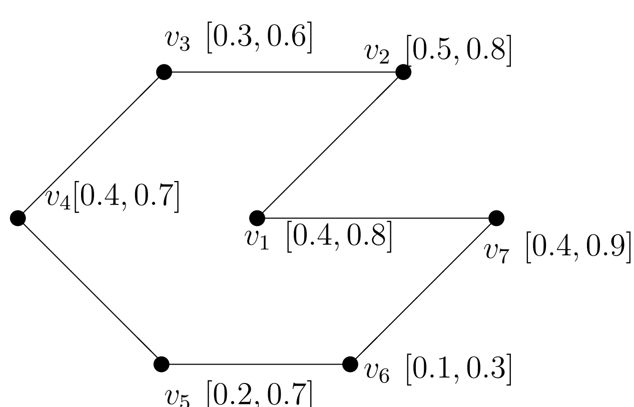

Fig 21

From the Fig (21), we see that $D_{tp} = \{v_2, v_3, v_4, v_5, v_6\}$ is minimal total perfect dominating set of G. Then $\gamma_{tp}(G) = 3.3$, $p = 4.75$ and $\Delta_N(G) = 1.5$. Now we have $p - \Delta_N(G) = 4.75 - 1.5 = 3.25$. Then $\gamma_{tp}(G) \geq p - \Delta_N(G)$. Thus $\gamma_{tp}(G) \leq p - \Delta_N(G) + 1$.

Remark 3.98 For any interval-valued fuzzy graph G has no isolated vertices, with $\gamma_{tp} - set$. Then

$$\frac{p}{\Delta_N(G) + 1} \leq \gamma_{tp}(G)$$

or

$$\frac{p}{\Delta_N(G) + 1} \geq \gamma_{tp}(G)$$

.

Example 3.99 Consider $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two interval-valued fuzzy graphs given in the following two Figurs, where all edges of G_1 and G_2 are effective, such that $A_1 = \{\frac{a}{[0.2,0.4]}, \frac{b}{[0.1,0.4]}, \frac{c}{[0.4,0.6]}, \frac{d}{[0.3,0.5]}, \frac{e}{[0.1,0.7]}, \frac{f}{[0.6,0.9]}, \frac{g}{[0.3,0.8]}, \frac{h}{[0.1,0.6]}, \frac{i}{[0.7,0.8]}, \frac{j}{[0.2,0.9]}, \frac{k}{[0.4,0.7]}\}$ and $A_2 = \{\frac{x}{[0.1,0.4]}, \frac{y}{[0.3,0.5]}, \frac{z}{[0.1,0.3]}, \frac{w}{[0.3,0.6]}, \frac{u}{[0.2,0.3]}, \frac{v}{[0.5,0.9]}, \frac{m}{[0.6,0.8]}, \frac{n}{[0.4,0.6]}\}$.

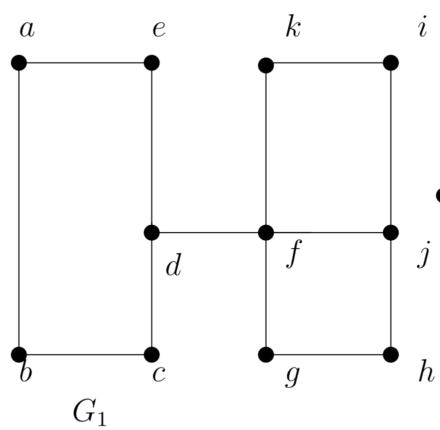

Fig 22

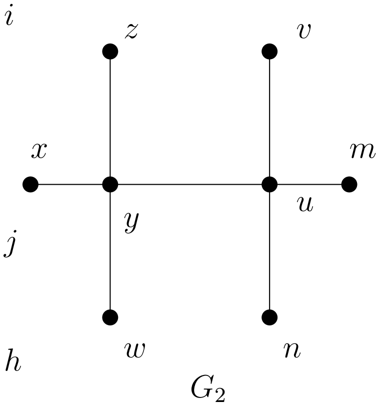

Fig 23

From the Fig (22), we see that a vertex subset

$D_1 = \{a, b, f, j\}$ is minimum total perfect dominating set of G. Then $\gamma_{tp}(G) = 2.75$,

$p = 7.45$ and $\Delta_N(G) = \{d, g, j, k\} = 2.85$. Hence $\frac{p}{\Delta_N(G)+1} = \frac{7.45}{2.85+1} = 1.935$. So

$$\frac{p}{\Delta_N(G)+1} \leq \gamma_{tp}(G)$$

.

Also from the Fig (23), we see that a vertex subset

$D_2 = \{y, u\}$ is minimum total perfect dominating set of G. Then $\gamma_{tp}(G) = 1.15$, $p = 4.95$ and $\Delta_N(G) = \{y, v, m, n\} = 2.5$. Hence $\frac{p}{\Delta_N(G)+1} = \frac{4.95}{2.5+1} = 1.4143$. So

$$\frac{p}{\Delta_N(G)+1} \geq \gamma_{tp}(G)$$

.

Theorem 3.100 Let G be any connected interval-valued fuzzy graph, with $\gamma_{tp} - set$ and $max(d_N(v))$. Then

$$\frac{p}{2(\Delta_N+1)} \leq \gamma_{tp}(G) \leq 2q - p + 1$$

.

proof 3.101 Let G be any connected interval-valued fuzzy graph, with $\gamma_{tp} - set$ and let v be a vertex of G with $\Delta_N(u) = d_N(v)$. Firstly we prove the lower bound. By the above Remark

$$\frac{p}{\Delta_N(G)+1} \leq \gamma_{tp}(G)$$

or

$$\frac{p}{\Delta_N(G)+1} \geq \gamma_{tp}(G)$$

, for any interval-valued fuzzy graph G has no isolated vertices. Then if

$$\frac{p}{\Delta_N(G)+1} \leq \gamma_{tp}(G)$$

, it is trivial $\Rightarrow \frac{p}{2(\Delta_N+1)} \leq \gamma_{tp}(G)$ or if

$$\frac{p}{\Delta_N(G)+1} \geq \gamma_{tp}(G)$$

, clear that also $\frac{p}{2(\Delta_N+1)} \leq \gamma_{tp}(G)$ .... (1). secondly we consider the upper bound. Since $\gamma_{tp} \leq p \Rightarrow \gamma_{tp} \leq 2(p-1) - p + 1 \Rightarrow \gamma_{tp} \leq 2q - p + 1$...........(2). Therefore, from (1) and (2) we get

$$\frac{p}{2(\Delta_N+1)} \leq \gamma_{tp}(G) \leq 2q - p + 1$$

.

29

Example 3.102 For the interval-valued fuzzy graph G given in the Fig (24), where every edge of G is strong edge.

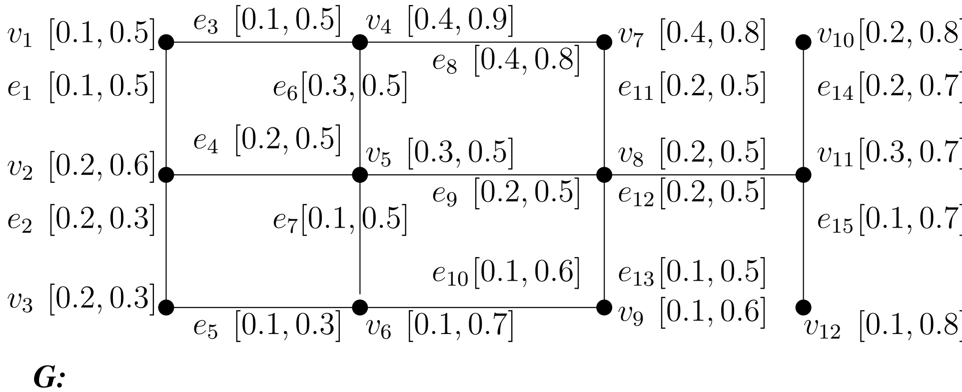

Fig 24

From the above Fig, we see that a vertex subset $D_{tp} = \{v_2, v_5, v_8, v_{11}\}$ is minimum total perfect dominating set of an interval-valued fuzzy graph G. Then $\gamma_{tp} = 2.65$, $p = 8.5$, $q = 10.15$ $\Rightarrow 2q = 20.3$, $max(d_N(v)) = v_5 \Rightarrow \Delta_N(G) = 2.9$. Thus $\frac{p}{2(\Delta_N+1)} = \frac{8.5}{2(2.9+1)} = 1.0897$ and $2q - p + 1 = 20.3 - 8.5 + 1 = 12.8$. Now we have $1.0897 \leq 2.65 \leq 12.8$. Hence

$$\frac{p}{2(\Delta_N + 1)} \leq \gamma_{tp}(G) \leq 2q - p + 1$$

.

4 Conclusion

In this paper, the concepts of total perfect dominating set and total Perfect domination number were defined on interval-valued fuzzy graphs and applied for the various types of interval-valued fuzzy graphs with suitable examples. The bounds of $\gamma_{tp}(G)$ and of Γ_{tp} were obtained with the known parameters in interval-valued fuzzy graphs. Further, we discussed many properties between the concept of total perfect domination and other concepts related to the interval-valued fuzzy graphs.

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