Tilahun A Muche

and

Willie Reynolds

Authors Affiliation:

Savannah State University

Department of Mathematics

Postal address:

P.O.Box 20047

3219 College Street

Savannah, GA, 31404

USA

Telephone:

Office: (912)-358-4305(mentor)

Cell: (404)-840-2233 (student)

Fax: (912)358-4768

email:

muchet@savannahstate.edu

and

wreynol1@student.savannahstate.edu

[1] *Tilahun Muche, Department of Mathematics, Savannah State University,GA*

[2] *Willie Reynolds, Department of Mathematics, Savannah State University,GA*

[1] [2]

THE DERIVATIVE OF FUNCTIONS AND THE CEILING DETERMINANT OF A SQUARE MATRIX

Abstract

Derivatives in calculus and Determinants of square matrices of any size in linear algebra are extensively used in STEM, Financial and Business fields to mention some. In this research paper, we develop mathematical formula to drill determinants for evaluating the derivatives of a function of higher order n. First, we introduce the Ceiling Determinant of a two by two matrix and extended this definition to find the derivative of a function of n order. This new approach will help students to easily memorize the formula and apply to their major academic discipline. We have application examples on complex variable and geometry.

Key words: Derivatives, Determinants, Ceiling Determinants, Complex Numbers.

Introduction

What is a Derivative of a function f?

The idea of the derivative of a function :

First we can tell what the idea of a derivative is. But the issue of computing derivatives is another thing entirely: a person can understand the idea without being able to effectively compute, and vice-versa. Suppose that f is a function of interest for some reason.We can give f some sort of geometric life by thinking about the set of points (x, y) so that $f(x) = y$. We would say that this describes a curve in the (x, y) plane.(And sometimes we think of x as moving "from left to right", imparting further intuitive or physical content to the story). For some particular number x_0 , let y_0 be the value $f(x_0)$ obtained as output by plugging x_0 into f as input. The point (x_0, y_0) is the point on our curve. The **tangent line** to the curve at the point (x_0, y_0) is the line passing through (x_0, y_0) and flat against the curve. The idea of the derivative $f^{'}(x_0)$ is that it is the slope of the tangent line at x_0 to the curve.

[1]mentor: Tilahun Muche, email: muchet@savannahstate.edu, the PSLSAMP Summer Scholarship Research Grant

[2]student: Willie Reynolds, email: wreynol1@student.savannahstate.edu

The derivative of a function f at a point x , written $\frac{dy}{dx} = f^{'}(x)$, is given by

$$lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right) = f^{'}(x)$$

if the limit exists.

Rules of Derivative :

Power Rule - For any real number n and functions of the form $f(x) = x^n$, the derivative of $f(x)$ is the product of the exponent n times x with the exponent reduced by 1.

$$\frac{dy}{dx} = f^{'}(x) = nx^{n-1}$$

Extended Product Rule : A formula used to find the derivative of the product of two or more functions f, g and h.

Given: (a) $y = (fg)(x)$

$$\frac{d(fg)(x)}{dx} = \frac{df}{dx}g(x) + \frac{dg}{dx}f(x)$$

$$= f'(x)g(x) + g'(x)h(x)$$

(b) $y = (fgh)(x)$

$$\frac{d(fgh)(x)}{dx} = \frac{df}{dx}(g(x)h(x)) + \frac{dg}{dx}(f(x)h(x)) + \frac{dh}{dx}(f(x)g(x))$$

$$= f'(x)(gh(x)) + g'(x)((fg)(x)) + h'(x)((fg)(x))$$

Remark: The nth derivative of a function $y = f(x)$ is denoted by $\frac{d^n y}{dx^n} = f^n(x)$

Example : Given $f(x) = x^2cos(x)$. Then

$$\frac{dy}{dx} = f^{'}(x) = 2xcosx - x^2sinx$$

$$\frac{d^2y}{dx^2} = f^{''}(x) = -x^2cosx - 4xsinx + 2cosx$$

$$\frac{d^3y}{dx^3} = f^{'''}(x) = x^2sinx - 6xcosx - 6sinx$$

$$\vdots$$

$$\frac{d^ny}{dx^n} = f^n(x)$$

Quotient Rule : A method of finding the derivative of a function that is the ratio of two differentiable functions.

Given: $y = f(x) = (\frac{g}{h})(x)$. Then $\frac{dy}{dx} = \frac{d(\frac{g(x)}{h(x)})}{d(x)} = f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{h^2(x)}, h(x) \neq 0$

Example: Given $f(x) = \frac{2x^2}{3sin(x)}$. Find $f'(x)$.

Solution : $f'(x) = \frac{(4x)(3sin(x)) - 2x^2(3cos(x))}{(3sin(x))^2} = \frac{12xsin(x) - 6x^2cos(x)}{9sin^2(x)} = \frac{12xsin(x)}{9sin^2(x)}$ - $\frac{6x^2cos(x)}{9sin^2(x)}$

$$= \frac{4}{3}xcsc(x) - \frac{2}{3}x^2cot(x)csc(x)$$

Definition: A matrix is a rectangular array of numbers. A matrix with m rows and n columns is said to have dimension $m \times n$ and may be represented as follows.

$$A_{mn} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$= (a_{ij}) \in R^{m \times n}$

Matrices and operations in Matrices

Operation in matrices :

Let A and B be two matrices of the same size. $A = (a_{ij})_{mxn}$ and $b = (b_{ij})_{mxn}$.

1. Addition: $A + B = (a_{ij} + b_{ij})_{mxn}$
2. Scalar Multiplication: $\alpha(A) = \alpha(a_{ij})_{mxn}$
3. Product: If A and B are matrices with $A = (a_{ij})_{mxn}$ and $B = (b_{ij})_{nxp}$ then $AB = C$ where $C = (c_{ij})_{mxp}$
4. Transpose: The transpose of an m by n matrix A is the n by m matrix A^t formed by turning rows into columns and vice versa.

Determinant : What is a Determinant ?

- A number resulted from a special combination of the numbers in a matrix.
- Every square matrix has a determinant.

Definition : Given a 2 by 2 matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{0.0.1}$$

The determinant of A denoted by $|A|$ or $Det(A)$ is calculated as

$$|A| = Det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12} \tag{0.0.2}$$

Definition : Ceiling Determinat

Given a square Matrix A of size n by n

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

The ceiling determinat of A denoted by $CeilDet(A)$ is defined as

$$CeilDet(A) = \left(\left\| \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{matrix} \right\| \right) \tag{0.0.3}$$

Example : Given

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{0.0.4}$$

Find the CeilDet(A).

Solution :

$$CeilDet(A) = \left(\left\| \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix} \right\| \right) = a_{11}a_{22} + a_{21}a_{12} \tag{0.0.5}$$

Application Problems

Example : **PRODUCT RULE**

Definition : Given two function f and g such that $f^{'}$ and $g^{'}$ exists . Denote $A = (fg) = f \times g$ and

$$D = \begin{pmatrix} f & g \\ f^{'} & g' \end{pmatrix} \tag{0.0.6}$$

$$Then\ A' = (fg)' = \ CeilDet(D) \ = \left(\left\| \begin{matrix} f & g \\ f' & g' \end{matrix} \right\| \right) = fg' + g'f \tag{0.0.7}$$

Example : Given $f(x) = x^2$ and $g(x) = ln(x)$. Find $(fg)'$.

Solution :

$$D = \begin{pmatrix} x^2 & (ln(x)) \\ (2x) & \frac{1}{x} \end{pmatrix} \tag{0.0.8}$$

Then $A' = (fg)'$

$$CeilDet(D) = \left(\left\| \begin{matrix} x^2 & ln(x) \\ 2x & \frac{1}{x} \end{matrix} \right\| \right) = x^2(\frac{1}{x}) + ln(x)(2x) = x + 2x(ln(x)) \tag{0.0.9}$$

Theorem : Given f, g and h differentiable functions. Prove that $(f \star g \star h)^{'} = f^{'} \star (g \star h)$ $+ \, g^{'} \star (f \star h) +$ $h^{'} \star (f \star g)$.

Proof : $(f \star g \star h) = (f \star g) \star h$ and $(f \star g \star h)^{'} = ((f \star g) \star h)^{'}$.

$$D = \begin{pmatrix} (f \star g) & h \\ (f \star g)^{'} & h^{'} \end{pmatrix} \tag{0.0.10}$$

Then $A' = ((f \star g) \star h)^{'} =$

$$CeilDet(D) = \left(\left\| \begin{matrix} (f \star g) & h \\ (f \star g)^{'} & h^{'} \end{matrix} \right\| \right) = (f \star g) \star h^{'} + h \star (f \star g)^{'} \tag{0.0.11}$$

From (0.0.11) above, consider $h \star (f \star g)^{'}$. Let

$$D = \begin{pmatrix} f & g \\ f^{'} & g^{'} \end{pmatrix} \tag{0.0.12}$$

Then $B' = (f \star g)' =$

$$CeilDet(D) = \left(\left\| \begin{matrix} f & g \\ f^{'} & g^{'} \end{matrix} \right\| \right) = f \star g^{'} + g \star f^{'} \tag{0.0.13}$$

Combing (0.0.11) and (0.0.13) we have,

$$D = \begin{pmatrix} (f \star g) & h \\ (f \star g)^{'} & h^{'} \end{pmatrix} \tag{0.0.14}$$

Then $A' = ((f \star g) \star h)^{'}$

$$CeilDet(D) = \left(\left\| \begin{matrix} (f \star g) & h \\ (f \star g)^{'} & h^{'} \end{matrix} \right\| \right) = (f \star g) \star h^{'} + h \star B^{'} = (f \star g) \star h^{'} + h \star (f \star g^{'} + g \star f^{'}) \tag{0.0.15}$$

$$= (f \star g) \star h^{'} + (h \star f) \star g^{'} + (h \star g) \star f^{'})$$

Definition : Given two functions f and g such that $f^{'}$ and g' exists. Then if

$$D = \begin{pmatrix} f & g \\ f^{'} & g^{'} \end{pmatrix} \tag{0.0.16}$$

the determinant of D, Denoted by $|D|$ is defined as

$$|D| = \begin{vmatrix} f & g \\ f^{'} & g^{'} \end{vmatrix} = fg^{'} - g^{'}f \tag{0.0.17}$$

Theorem:

Let $B = (\frac{f}{g})$. Then $B' = (\frac{f}{g})^{'} = (\frac{1}{g^2}) \star |D|$

Proof : $(\frac{f}{g})^{'} = \frac{f^{'}g - g^{'}f}{g^2} = (\frac{1}{g^2})(f^{'}g - g^{'}f)$. This implies

$$(\frac{f}{g})^{'} = \frac{1}{g^2} \star \begin{vmatrix} f & g \\ f^{'} & g^{'} \end{vmatrix} = (\frac{1}{g^2}) \star |D| \tag{0.0.18}$$

Pascal triangle

n=0:					1					
n=1:				1		1				
n=2:			1		2		1			
n=3:		1		3		3		1		
n=4:	1		4		6		4		1	

Binomial Expansion

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Higher order Derivatives and Ceiling Determinant :

Theorem : Given $(f\star g) = f\star g$. The nth derivative of $(f\star g)$, that is $(f\star g)^n = \frac{d^n((fg)(x))}{dx^n} = (fg)^{(n)} = \sum_{i=0}^{n-1} \binom{n-1}{i} \left(\begin{Vmatrix} f^{(i)} & g^{(i)} \\ f^{(n-i)} & g^{(n-i)} \end{Vmatrix} \right) = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k)}$

Proof. The proof follows induction principle.

$$\frac{d(f\star g)}{dx} = (f\star g)^{'} = CeilDet(D) = \left(\begin{Vmatrix} f & g \\ f^{'} & g^{'} \end{Vmatrix} \right) = f^{'}\star g + g^{'}\star f \tag{0.0.19}$$

$$\frac{d^2(f\star g)}{dx} = (f\star g)^{''} = \left(\begin{Vmatrix} f & g \\ f^{''} & g^{''} \end{Vmatrix} \right) + \left(\begin{Vmatrix} f^{'} & g^{'} \\ f^{'} & g^{'} \end{Vmatrix} \right) = fg^{''} + 2f^{'}g^{'} + f^{''}g \tag{0.0.20}$$

$$\frac{d^3(f\star g)}{dx} = (f\star g)^{'''} = \left(\begin{Vmatrix} f & g \\ f^{'''} & g^{'''} \end{Vmatrix} \right) + 2\left(\begin{Vmatrix} f^{'} & g^{'} \\ f^{''} & g^{''} \end{Vmatrix} \right) + \left(\begin{Vmatrix} f^{''} & g^{''} \\ f^{'} & g^{'} \end{Vmatrix} \right) \tag{0.0.21}$$

$$= fg^{'''} + 3f^{''}g^{'} + 3f^{'}g^{''} + f^{'''}g$$

$$\vdots$$

$$\frac{d^n((fg)(x))}{dx^n} = (fg)^{(n)} = \binom{n-1}{0} \left(\begin{Vmatrix} f & g \\ f^{(n)} & g^{(n)} \end{Vmatrix} \right) + \binom{n-1}{1} \left(\begin{Vmatrix} f' & g' \\ f^{(n-1)} & g^{(n-1)} \end{Vmatrix} \right)$$

$$+\binom{n-1}{2} \left(\begin{Vmatrix} f'' & g'' \\ & \\ f^{(n-2)} & g^{(n-2)} \end{Vmatrix} \right) + \cdots + \binom{n-1}{n-1} \left(\begin{Vmatrix} f^{(n-1)} & g^{(n-1)} \\ f' & g' \end{Vmatrix} \right)$$

$$= \sum_{i=0}^{n-1} \binom{n-1}{i} \left(\begin{Vmatrix} f^{(i)} & g^{(i)} \\ f^{(n-i)} & g^{(n-i)} \end{Vmatrix} \right) = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k)}$$

$$where\ fand\ g\ are\ n\ times\ differentiable\ function,$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \; and \; \binom{m}{k-1} + \binom{m}{k} = \binom{m+1}{k}.$$

$$(fg)^{(n+1)} = \left((fg)^{(n)}\right)'$$

$$(fg)^{(n+1)} = \left(\sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k)}\right)'$$

$$\sum_{k=0}^{n} \binom{n}{k} \left(f^{(n-k)} g^{(k)}\right)' = \sum_{k=0}^{n} \binom{n}{k} \left(f^{(n-k+1)} g^{(k)} + f^{(n-k)} g^{(k+1)}\right) =$$

$$\sum_{k=0}^{n} \binom{n}{k} f^{(n-k+1)} g^{(k)} + \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k+1)} =$$

$$\sum_{k=0}^{n} \binom{n}{k} f^{(n-k+1)} g^{(k)} + \sum_{k=1}^{n+1} \binom{n}{k-1} f^{(n-(k-1))} g^{((k-1)+1)} =$$

$$\binom{0}{0} f^{(n+1)} f^{(0)} + \sum_{k=1}^{n} \binom{n}{k} f^{(n+1-k)} g^{(k)} + \binom{n}{n} f^{(0)} g^{(n+1)} +$$

$$\sum_{k=1}^{n} \binom{n}{k-1} f^{(n+1-k)} g^{(k)} = \binom{0}{0} f^{(n+1)} g^{(0)} + \binom{n+1}{n+1} f^{(0)} g^{(n+1)} + \sum_{k=1}^{n} \left(\binom{n}{k} + \binom{n}{k-1}\right) f^{(n+1-k)} g^{(k)} \tag{0.0.22}$$

$$(fg)^{(n+1)} = \binom{0}{0} f^{(n+1)} g^{(0)} + \binom{n+1}{n+1} f^{(0)} g^{(n+1)} + \sum_{k=1}^{n} \binom{n+1}{k} f^{(n+1-k)} g^{(k)} =$$

$$\sum_{k=0}^{n+1} \binom{n+1}{k} f^{(n+1-k)} g^{(k)}. \tag{0.0.23}$$

$$(fg)^{m+1}(x) = \left(\sum_{k=0}^{m} \binom{m}{k} f^{(k)}(x) g^{(m-k)}(x)\right)'$$

$$= \left(\sum_{k=0}^{m} \binom{m}{k} f^{(k+1)}(x) g^{(m-k)}(x)\right) + \left(\sum_{k=0}^{m} \binom{m}{k} f^{(k)}(x) g^{(m-k+1)}(x)\right)$$

$$= f^{(m+1)}(x)g(x) + \left(\sum_{k=0}^{m-1} \binom{m}{k} f^{(k+1)}(x) g^{(m-k)}(x)\right) + \left(\sum_{k=1}^{m} \binom{m}{k} f^{(k)}(x) g^{(m-k+1)}(x)\right) + f(x)g^{(m+1)}(x)$$

$$= f^{(m+1)}(x)g(x) + \left(\sum_{k=0}^{m-1} \binom{m}{k} f^{(k+1)}(x) g^{(m-k)}(x)\right) +$$

$$\left(\sum_{k=0}^{m-1} \binom{m}{k+1} f^{(k+1)}(x) g^{(m-k)}(x)\right) + f(x)g^{(m+1)}(x). \tag{0.0.24}$$

Derivative of composition of Functions :

Remark : $f = f \circ I$, where I is the identity function $I(x) = x$.

Definition : $\frac{df}{dx} = \frac{d(f \circ I)}{dx} =$

Let

$$D = \begin{pmatrix} f & I \\ f' & 0 \end{pmatrix} \tag{0.0.25}$$

$$\frac{df}{dx} = \frac{d(f \circ I)}{dx} = CeilDet(D) = \left(\begin{Vmatrix} f & I \\ f' & 0 \end{Vmatrix} \right) = f'(I(x)) \tag{0.0.26}$$

Theorem : $(f \circ g)(x) = f((g \circ I)(x))$ and $((f \circ g)(x))' = (f((g \circ I)(x)))' = f'(g(x)) \star (g'(x))$.

Proof :

$$(f \circ g)' = \left(\begin{Vmatrix} f & I \\ f' & 0 \end{Vmatrix} \right)(g(x)) \star \left(\begin{Vmatrix} g & g' \\ I & 0 \end{Vmatrix} \right)(x) = f'(g(x)) \star g'(x) \tag{0.0.27}$$

Application Problem

Theorem : In a plane, a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) has area

$$\sum_{i=2k-1} \frac{1}{2} \left(\left\| \begin{matrix} x_i & x_{i+1} \\ y_{i+2} & y_{i+1} \end{matrix} \right\| \right) - \sum_{i=2k-1} \frac{1}{2} \left(\left\| \begin{matrix} y_i & y_{i+1} \\ x_{i+2} & x_{i+1} \end{matrix} \right\| \right) \tag{0.0.28}$$

where $x_4 = x_1$ and $y_4 = y_1$ and $y_i = 0$ for $i \geq 5$.

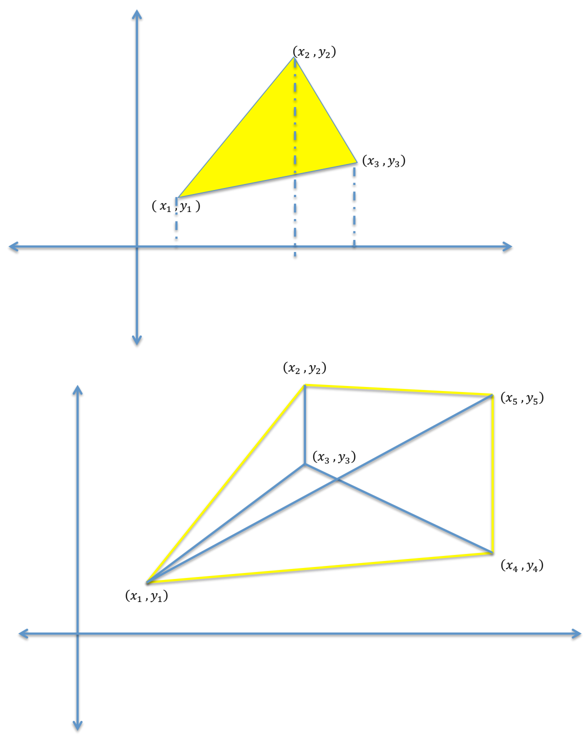

Figure 1: A triangle and a quadrilateral with labelled vertices

Proof

A triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) has area

$$A = \frac{1}{2} \star Det \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \tag{0.0.29}$$

The vector from (x_3, y_3) to (x_2, y_2) is $(x_3 - x_2, y_3 - y_2)$, and similarly for the vector from (x_3, y_3) to (x_1, y_1) it is $(x_3 - x_1, y_3 - y_1)$. It is known that subtracting rows does not change the determinant. We effectively move the triangle along so that one of its vertices is at the origin. Now

$$D = \begin{pmatrix} x_1 - x_3 & y_1 - y_3 & 0 \\ x_2 - x_3 & y_2 - y_3 & 0 \\ x_3 & y_3 & 1 \end{pmatrix} \tag{0.0.30}$$

and Determinant of matrix D can be found by cofactor expansion along the third column and one half of the determinant gives the area.

Area $= \frac{1}{2}$ $[(x_1 - x_3)(y_2 - y_3)$ - $(y_1 - y_3)(x_2 - x_3)]$

$= \frac{1}{2}$ $[x_1y_2 - x_1y_3 - x_3y_2 + x_3y_3 - y_1x_2 + y_1x_3 + y_3x_2 - y_3x_3]$

$= \frac{1}{2}$ $[x_1y_2 - x_1y_3 + y_1x_2 + y_1x_3 - y_3x_2]$

$=$

$$\sum_{i=2k-1} \frac{1}{2}\left(\begin{Vmatrix} x_i & x_{i+1} \\ y_{i+2} & y_{i+1} \end{Vmatrix}\right) - \sum_{i=2k-1} \frac{1}{2}\left(\begin{Vmatrix} y_i & y_{i+1} \\ x_{i+2} & x_{i+1} \end{Vmatrix}\right) \tag{0.0.31}$$

where $x_4 = x_1$ and $y_4 = y_1$ and $y_i = 0$ for $i \geq 5$.

Theorem : In a plane, a triangle with corners (x_1, y_1), (x_2, y_2) and (x_3, y_3) $\ldots(x_n, y_n)$ has area

$$\sum_{i=2k-1} \frac{1}{2}\left(\begin{Vmatrix} x_i & x_{i+1} \\ y_{i+2} & y_{i+1} \end{Vmatrix}\right) - \sum_{i=2k-1} \frac{1}{2}\left(\begin{Vmatrix} y_i & y_{i+1} \\ x_{i+2} & x_{i+1} \end{Vmatrix}\right) \tag{0.0.32}$$

where $x_n = x_1$ and $y_n = y_1$ and $y_{i+2} = 0$ for $i \geq n + 2$.

Example : Find the area of a triangle whose vertices are $(1,0)$, $(2,2)$ and $(4,3)$.
Solution : Let the vertices $(x_1, y_1) = (1,0)$, $(x_2, y_2) = (2,2)$ and $(x_3, y_3) = (4,3)$

$$Area = \frac{1}{2}\left(\begin{Vmatrix} x_1 & x_2 \\ y_3 & y_2 \end{Vmatrix}\right) + \frac{1}{2}\left(\begin{Vmatrix} x_3 & x_4 \\ y_5 & y_4 \end{Vmatrix}\right) - \frac{1}{2}\left(\begin{Vmatrix} y_1 & y_2 \\ x_3 & x_2 \end{Vmatrix}\right) - \frac{1}{2}\left(\begin{Vmatrix} y_3 & y_4 \\ x_5 & x_4 \end{Vmatrix}\right) \tag{0.0.33}$$

$=$

$$Area = \frac{1}{2}\left(\begin{Vmatrix} 1 & 2 \\ 3 & 2 \end{Vmatrix}\right) + \frac{1}{2}\left(\begin{Vmatrix} 4 & 1 \\ 0 & 0 \end{Vmatrix}\right) - \frac{1}{2}\left(\begin{Vmatrix} 0 & 2 \\ 4 & 2 \end{Vmatrix}\right) - \frac{1}{2}\left(\begin{Vmatrix} 3 & 0 \\ 0 & 1 \end{Vmatrix}\right) = \frac{3}{2} \tag{0.0.34}$$

Theorem : In a plane, points (x_1, y_1), (x_2, y_2) and $(x_3, y_3) \ldots (x_n, y_n)$ are collinear

$$\sum_{i=2k-1}\left(\begin{Vmatrix} x_i & x_{i+1} \\ y_{i+2} & y_{i+1} \end{Vmatrix}\right) - \sum_{i=2k-1}\left(\begin{Vmatrix} y_i & y_{i+1} \\ x_{i+2} & x_{i+1} \end{Vmatrix}\right) = 0] \tag{0.0.35}$$

Complex Number

A matrix, Z, whose elements are complex numbers can be written $[Z_{ij}]$,where $Z_{ij} = X_{ij}+Y_{ij}i$ or $Z = X + iY$, where "i" is the notation for $\sqrt{-1}$.
The latter form shows a separation of the real and imaginary parts into separate matrices. In this notation, both X and Y, are composed of real numbers. A matrix, $W = X - iY$, is called the "conjugate" of Z. The transpose of W is referred to as the "associate" of Z.

The sum, or product, of two complex matrices can be formed in the straightforward, element by element, way, using complex arithmetic, or using the second notation, $(Z = X + iY)$, previously coded (real arithmetic) routines can be used, since X and Y are composed of real numbers.

Theorem : Given $Z_1 = X_1 + iY_1$ and $Z_2 = X_2 + iY_2$. Then

$$Z_1 Z_2 = \begin{vmatrix} X_1 & Y_1 \\ Y_2 & X_2 \end{vmatrix} + i \left(\begin{Vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{Vmatrix} \right) \tag{0.0.36}$$

Proof :

Given $Z_1 = X_1 + iY_1$ and $Z_2 = X_2 + iY_2$. This implies,

$$|Z_1 Z_2| = |\left(X_1 + iY_1\right)\left(X_2 + iY_2\right)|$$

$$= X_1X_2 + iX_1Y_2 + iY_1X_2 - Y_1Y_2$$

$$= X_1X_2 - Y_1Y_2 + iX_1Y_2 + iY_1X_2$$

$$= X_1X_2 - Y_1Y_2 + i\left(X_1Y_2 + Y_1X_2\right)$$

$$= \begin{vmatrix} X_1 & Y_1 \\ Y_2 & X_2 \end{vmatrix} + i \left(\begin{Vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{Vmatrix} \right) \tag{0.0.37}$$

CONCLUSION:

Consider a triangle with vertices at (x_1, y_1), (x_2, y_2), and (x_3, y_3). If the triangle were a right triangle, it would be pretty easy to compute the area of the triangle by finding one-half the product of the base and the height. However, when the triangle is not a right triangle, there are a couple of other ways that the area can be found, Herons Formula, geometric techniques and determinants.The following are lists of some applications of matrix and determinant: Cryptography, encode and decode messages, classical and really concrete example would be a discrete Markov chain, and optimization. In this research paper we introduce notation of ceiling determinant with some application. We are working to extend properties of determinant to develop an algorithm and also to develop a formula for calculating volumes, vectors and matrix differentiation, and financial problems.

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