Cubic structures applied to ideals of co-residuated lattices

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Abstract

The concept of cubic ideals in co-residuated lattices is introduced and some interesting properties are obtained. Characterization theorem of cubic ideals is also discussed by the notion of cubic level sets. We construct Cartesian product of two cubic ideals by using max-min operations, and give some characterizations of them.

Keywords: co-residuated lattice; cubic ideal; Cartesian product

1. Introduction

It is well known that the study of multivalued logic systems and corresponding algebraic systems is closely related. MV-algebras were introduced by Chang [1] 1958 as to provide an algebraic proof of the completeness theorem of infinite valued Łukasiewicz propositional calculus. The theory of residual lattices based on triangular modules is important for providing an algebra frame for the algebraic semantics of formal fuzzy logics such as MV-algebras, BL-algebras and BCK-algebras. MV-algebras is characterized by the operator $\oplus$ which could be looked as the generation of conorm in lattices. In the properties of MV-algebras, there existed an operator $\ominus$ such that $a \leq b \oplus c$ if and only if $a \ominus c \leq b$, and $(\oplus, \ominus)$ constituted a pair just like adjoint pair $(\otimes, \rightarrow)$. Therefore, Zheng and Wang [2] proposed the notion of coadjoint pair, and coresiduated lattice, as the dual algebra structure of residuated lattice. Coresiduated lattices provide a new frame for logic algebras, Zheng and Wang pointed out that the notion of normal coresiduated lattices is consistency with that of normal residuated lattices. Based on the properties of co-residuated, Zhu [3] proved that regular co-residuated lattices are equivalent to involutory BCK-lattices.

Since ideals are closely related to congruence relations with which one can associate quotient algebras, so the ideal theory is a very effectively tool to study logical algebras and the completeness of the corresponding nonclassical logics In the theory of MV-algebras, as in various algebraic structures, the focus is the notion of ideals. The properties of ideals in coresiduated lattices were investigated and the embedding theorem of coresiduated lattices was obtained in [4]. In addition, based on the fuzzy set theory introduced by Zadeh, the related fuzzy structures (i.e., the fuzzification) of ideals in in various logic algebras have captured many scholars attention. Using falling shadows theory, [5] proposed the concept of falling fuzzy (implicative) ideals which as a generalization of a $T_\wedge$-fuzzy (implicative) ideal in MV-algebras. Al-Masarwah and Ahmad introduced the notion of doubt bipolar fuzzy H-ideals of BCK/BCI-algebras and investigate some interesting properties [6]. The notion of fuzzy ideals are introduced in coresiduated lattices in [7], and the characterizations of fuzzy ideals, fuzzy prime ideals, and fuzzy strong prime ideals in coresiduated lattices are investigated and also some relations between ideals and fuzzy ideals are established. Using a fuzzy set and an interval-valued fuzzy set, Jun et al. [8] introduced a new notion, called a cubic set, and investigated several properties, then they applied the cubic theory to BCK/BCI-algebras, and proposed cubic P-ideals and cubic α-ideals [9]. Continue the Jun's work, Senapati and Shum applied the concept of cubic sets to implicative ideals of BCKBCK-algebras, and then discussed relations among cubic implicative ideals, cubic subalgebras and cubic ideals of BCK-algebras [10].

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In the paper, we apply cubic sets to co-residuated lattices, and introduce the notion of cubic ideals. Then some characterizations of cubic ideals are discussed. Particularly, the notion of Cartesian product of two cubic ideals by using max-min operations is introduced, and some related properties are studied.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions which are necessary for this paper.

Definition 2.1. [2] *Let P be a poset, and $\oplus, \ominus : P \times P \to P$ be two binary operations. $(\oplus, \ominus)$ is called a coadjoint pair on P if it satisfies the following conditions:*

(1) *$\oplus$ is isotone;*
(2) *$\ominus$ is isotone on first variable and antitone on second variable;*
(3) *$a \le b \oplus c$ if and only if $a \ominus c \le b$, for any $a, b, c \in P$.*

Definition 2.2. [2] *An algebra $(L, \vee, \wedge, \oplus, \ominus, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ is called a co-residuated lattice if it satisfies the following axioms:*

(i) *$(L, \wedge, \vee, 0, 1)$ is a bounded lattice with the greatest element 1 and the smallest element 0,*
(ii) *$(L, \oplus, 0)$ is a commutative monoid,*
(iii) *$(\oplus, \ominus)$ is a coadjoint on L.*

In what follows, $(L, \vee, \wedge, \oplus, \ominus, 0, 1)$ is always assumed to be a co-residuated lattice and will often be referred to by its support set L.

Proposition 2.3. ([2, 4]) *For any co-residuated lattice $(L, \vee, \wedge, \oplus, \ominus, 0, 1)$, we have: for any $x, y, z, w, v \in L$,*

(1) *$1 \oplus x = 1$, $0 \oplus x = x$, $x \ominus 0 = x$, $x \ominus y \le x \le x \oplus y$;*
(2) *$x \le y$ if and only if $x \ominus y = 0$, $x \oplus y = 0$ if and only if $x = y = 0$;*
(3) *$(x \oplus y) \ominus y \le x \le (x \ominus y) \oplus y$;*
(4) *$x \ominus (x \wedge y) = x \ominus y$, $(x \vee y) \ominus y = x \ominus y$;*
(5) *$x \ominus (x \ominus y) \le x \wedge y \le x \vee y \le (x \ominus y) \oplus y$;*
(6) *$(x \oplus y) \ominus (y \oplus z) \le x \ominus z \le (x \ominus y) \oplus (y \ominus z)$;*
(7) *$(x \ominus z) \ominus (y \ominus z) \le x \ominus y$, $(z \ominus y) \ominus (z \ominus x) \le x \ominus y$.*

A nonempty subset I of L is called an ideal if it satisfies: (i) $0 \in I$; (ii) for any $x, y \in L$, if $x \in I$ and $y \le x$, then $y \in I$; (iii) $x, y \in I$ implies $x \oplus y \in I$. It has been shown that a nonempty subset I of L is an ideal if and only if (i) $0 \in I$; (2) $x \in I$ and $y \ominus x \in I$ imply $y \in I$, for any $x, y \in L$

Let L_1 and L_2 be two co-residuated lattices. A function $f : L_1 \to L_2$ is a homomorphism if it satisfies the following conditions: satisfying $f(1_1) = 1_2$, $f(0_1) = 0_2$, $f(a * b) = f(a) \star f(b)$, where $* \in \{\vee_1, \wedge_1, \oplus_1, \ominus_1\}$ in L_1 and $\star \in \{\vee_2, \wedge_2, \oplus_2, \ominus_2\}$ in L_2.

The determination of maximum and minimum between two real numbers is very simple but it is not simple for two intervals. In [11] Biswas described a method to find max/sup and min/inf between two intervals or set of intervals. A closed subinterval $\tilde{a} = [a^-, a^+]$ of a closed unit interval $[0, 1]$ is called an interval number, where $0 \le a^- \le a^+ \le 1$. Denote by $D[0, 1]$ the set of all interval numbers. We define the operations $\wedge$, $\vee$, $\ge$, $\le$ and $=$ in case of two elements in $D[0, 1]$. Consider two elements $\tilde{a}_1 = [a_1^-, a_1^+], \tilde{a}_2 = [a_2^-, a_2^+]$ in $D[0, 1]$, then

(1) $\tilde{a}_1 \ge \tilde{a}_2$ if and only if $a_1^- \ge a_2^-$ and $a_1^+ \ge a_2^+$;
(2) $\tilde{a}_1 \le \tilde{a}_2$ if and only if $a_1^- \le a_2^-$ and $a_1^+ \le a_2^+$;
(3) $\tilde{a}_1 = \tilde{a}_2$ if and only if $a_1^- = a_2^-$ and $a_1^+ = a_2^+$;
(4) $\tilde{a}_1 \wedge \tilde{a}_2 = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}]$;
(5) $\tilde{a}_1 \vee \tilde{a}_2 = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]$;
(6) $\mathrm{rinf}_{i\in\Lambda}\tilde{a}_i = [\inf_{i\in\Lambda} a_i^-, \inf_{i\in\Lambda} a_i^+]$, where $\tilde{a}_i \in D[0, 1]$, $i \in \Lambda$;

(7) $\mathrm{rsup}_{i\in\Lambda}\tilde{a}_i = [\sup_{i\in\Lambda} a_i^-, \sup_{i\in\Lambda} a_i^+]$, where $\tilde{a}_i \in D[0,1]$, $i \in \Lambda$;

other operations $>$ and $<$ can be defined analogously.

An interval-valued fuzzy set A over a nonempty set X is an object having the form $A = \{(x, [\mu_A^-(x), \tilde{\mu}_A(x))|x \in X\}$, where $\tilde{\mu}_A : X \to D[0,1]$.

Based on fuzzy sets and interval-valued fuzzy sets, Jun et al. [8] introduced the notion of cubic sets, and investigated several properties.

Definition 2.4. *Let X be a nonempty set. A cubic set A in X as an object having the following form:*

$$A = \{(x, \tilde{\mu}_A(x), \lambda_A(x))|x \in X\},$$

where $\tilde{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set in X and λ_A is a fuzzy set in X, and $\mu_A^+(x)+\lambda_A(x) \leq 1$. In order to facilitate our subsequent discussion, a cubic set A is briefly denoted by $A = (\tilde{\mu}_A, \lambda_A)$, the number $A(x) = (\tilde{\mu}_A(x), \lambda_A(x))$ is called a cubic element, and we denote by $\mathbb{C}(X)$ the set of all cubic sets in X.

For two cubic elements $A(x)$ and $A(y)$ of the cubic set A, we give the following operations:

(1) $A(x) \preceq A(y)$ iff $\tilde{\mu}_A(x) \leq \tilde{\mu}_A(y)$, $\lambda_A(x) \geq \lambda_A(y)$;
(2) $A(x) \prec A(y)$ iff $\tilde{\mu}_A(x) < \tilde{\mu}_A(y)$, $\lambda_A(x) > \lambda_A(y)$;
(3) $A(x) \succeq A(y)$ iff $\tilde{\mu}_A(x) \geq \tilde{\mu}_A(y)$, $\lambda_A(x) \leq \lambda_A(y)$;
(4) $A(x) \succ A(y)$ iff $\tilde{\mu}_A(x) > \tilde{\mu}_A(y)$, $\lambda_A(x) < \lambda_A(y)$;
(5) $A(x) = A(y)$ iff $\tilde{\mu}_A(x) = \tilde{\mu}_A(y)$, $\lambda_A(x) = \lambda_A(y)$;
(6) $A(x) \veebar A(y) = (\tilde{\mu}_A(x) \vee \tilde{\mu}_A(y), \lambda_A(x) \wedge \lambda_A(y))$;
(7) $A(x) \barwedge A(y) = (\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(y), \lambda_A(x) \vee \lambda_A(y))$.

If $A_i = (\tilde{\mu}_{A_i}, \lambda_{A_i})$ $(i \in \Lambda)$ are cubic elements, where Λ is an index set, then we define:

$$\overline{\sup}_{i\in\Lambda} A_i = (\mathrm{rsup}_{i\in\Lambda}\tilde{\mu}_{A_i}, \inf_{i\in\Lambda} \lambda_{A_i}).$$

Let $A = (\tilde{\mu}_A, \lambda_A)$ and $B = (\tilde{\mu}_B, \lambda_B)$ be two cubic sets of X, we put $A \sqsubseteq B$ if and only if $A(x) \preceq B(x)$ for any $x \in X$; $A \sqsubset B$ if and only if $A(x) \prec B(x)$ for any $x \in X$.

3. Cubic ideals of co-residuated lattices

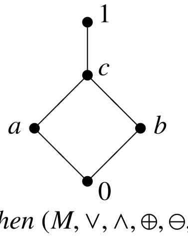
In this section, we give the notion of cubic ideals of co-residuated lattices and study several properties of them.

Definition 3.1. *Let $A = (\tilde{\mu}_A, \lambda_A)$ be a cubic set of a co-residuated lattice L. Then A is called a cubic ideal of L if it satisfies the following conditions: for any $x, y \in L$,*

(1) $A(x) \preceq A(0)$;
(2) $A(y) \barwedge A(x \ominus y) \preceq A(x)$.

The following example shows that cubic ideals exist.

Example 3.2. *Let $M = \{0, a, b, c, 1\}$ be a set with the Hasse diagram and Cayley tables as follows.*



$\oplus$	0	a	b	c	1
0	0	a	b	c	1
a	a	a	c	c	1
b	b	c	b	c	1
c	c	c	c	c	1
1	1	1	1	1	1

$\ominus$	0	a	b	c	1
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	b	a	0	0
1	1	1	1	1	0

Then $(M, \vee, \wedge, \oplus, \ominus, 0, 1)$ is a co-residuated lattice. Define a cubic set $A = (\tilde{\mu}_A, \lambda_A)$ in M as follows:

$$\tilde{\mu}_A(x) = \begin{cases} [0.8, 0.9], & x = 0, \\ [0.4, 0.6], & x = a, \\ [0.3, 0.7], & x = b, \\ [0.3, 0.6], & x = c, \\ [0.1, 0.4], & x = 1; \end{cases} \quad \lambda_A(x) = \begin{cases} 0.1, & x = 0, \\ 0.5, & x = a, \\ 0.4, & x = b, \\ 0.5, & x = c, \\ 0.7, & x = 1. \end{cases}$$

It is easy to check that A is a cubic ideal of L.

Proposition 3.3. *Let $A \in \mathbb{C}(L)$. Then A is a cubic ideal of L if and only if for any $x, y \in M$,*

(1) *$x \le y$ implies $A(y) \preceq A(x)$;*
(2) *$A(x) \,\bar{\wedge}\, A(y) \preceq A(x \oplus y)$.*

Proof. Suppose that A is a cubic ideal of L and $x, y \in L$, if $x \le y$, then $A(x) \succeq A(y) \bar{\wedge} A(x \ominus y) = A(y) \bar{\wedge} A(0) = A(y)$. From $(x \oplus y) \ominus y \le x$, we have $A((x \oplus y) \ominus y) \succeq A(x)$, and $A(x \oplus y) \succeq A(y) \,\bar{\wedge}\, A((x \oplus y) \ominus y) \succeq A(x) \,\bar{\wedge}\, A(y)$.

Conversely, for any $x \in L$, we have $0 \le x$, thus $A(x) \preceq A(0)$. For any $x, y \in L$, since $x \le (x \ominus y) \oplus y$, then $A((x \ominus y) \oplus y) \preceq A(x)$, it follows that $A(y) \,\bar{\wedge}\, A(x \ominus y) \preceq A((x \ominus y) \oplus y) \preceq A(x)$, hence A is a cubic ideal of L.

Let $A = (\tilde{\mu}_A, \lambda_A) \in \mathbb{C}(L)$, $r \in [0, 1]$ and $[s, t] \in D[0, 1]$ such that $r + t \le 1$. The set

$$L(A; ([s, t], r)) = \{x \in L | \tilde{\mu}_A(x) \ge [s, t], \lambda_A(x) \le r\}$$

is called a $([s, t], r)$-cubic level set of A. The proof of the next proposition is straightforward, and will be omitted.

Proposition 3.4. *Let $A \in \mathbb{C}(L)$. Then the following statements are equivalent:*

(1) *A is a cubic ideal of L;*
(2) *for any $r \in [0, 1]$ and $[s, t] \in D[0, 1]$ such that $r + t \le 1$, the nonempty cubic level set $L(A; ([s, t], r))$ is an ideal of L.*

Theorem 3.5. *Let $A \in \mathbb{C}(L)$. Then A is a cubic ideal of L if and only if $z \ominus x \le y$ implies $A(x) \bar{\wedge} A(y) \preceq A(z)$ for any $x, y, z \in L$.*

Proof. Assume that A is a cubic ideal of L and there exist $x, y, z \in M$ such that $z \ominus x \le y$, then $A(z \ominus x) \succeq A(y)$, it follows that $A(z) \succeq A(x) \,\bar{\wedge}\, A(z \ominus x) \succeq A(x) \,\bar{\wedge}\, A(y)$.

Conversely, from $0 \ominus x = 0 \le x$ we have $A(0) \succeq A(x) \,\bar{\wedge}\, A(x) = A(x)$. Since $x \ominus (x \ominus y) \le y$, then $A(x) \succeq A(y) \,\bar{\wedge}\, A(x \ominus y)$, and so A is a cubic ideal of L.

Proposition 3.6. *Let $A \in \mathbb{C}(L)$. Then A is a cubic ideal of L if and only if for any $x, y \in L$,*

(1) *$A(x) \,\bar{\wedge}\, A(y) \preceq A(x \oplus y)$,*
(2) *$A(y) \preceq A(x \wedge y)$.*

Proof. We only need to show that (2) is equivalent to (1) of Proposition 3.3. Assume that A is a cubic ideal of L, since $x \wedge y \le y$ for any $x, y \in L$, then $A(y) \preceq A(x \wedge y)$.

Conversely, suppose that (2) of Proposition 3.6 holds. For any $x, y \in L$, if $x \le y$, then $x \wedge y = x$, and hence $A(x) = A(x \wedge y) \succeq A(y)$, therefore (1) of Proposition 3.3 is valid.

Proposition 3.7. *Let A be a cubic ideal of L. Then the following results hold: for any $x, y, z \in L$,*

(1) *if $A(x \ominus y) = A(0)$, then $A(y) \preceq A(x)$;*
(2) *$A(x \vee y) = A(x) \,\bar{\wedge}\, A(y)$;*
(3) *$A(x \oplus y) = A(x) \,\bar{\wedge}\, A(y)$;*
(4) *$A(x \ominus y) \,\bar{\wedge}\, A(y \ominus z) \preceq A(x \ominus z)$.*

Proof. (1) Since A is a cubic ideal of L, then $A(y) = A(y) \,\bar{\wedge}\, A(0) = A(y) \,\bar{\wedge}\, A(x \ominus y) \preceq A(x)$ by Definition 3.1.

(2) Since $x \vee y \le x \oplus y$, according to Proposition 3.3, we have $A(x) \,\bar{\wedge}\, A(y) \preceq A(x \oplus y) \preceq A(x \vee y)$. As for the reverse inequality, from $x, y \le x \vee y$, we have $A(x \vee y) \preceq A(x)$ and $A(x \vee y) \preceq A(y)$, and so $A(x \vee y) \preceq A(x) \,\bar{\wedge}\, A(y)$, therefore (2) is valid.

(3) Since $x \vee y \le x \oplus y$, using Proposition 3.6 and (2) we get that $A(x) \,\bar{\wedge}\, A(y) \preceq A(x \oplus y) \preceq A(x \vee y) = A(x) \,\bar{\wedge}\, A(y)$, and so $A(x \oplus y) = A(x) \,\bar{\wedge}\, A(y)$.

(4) Notice that $x \ominus z \le (x \ominus y) \oplus (y \ominus z)$, we get that $A(x \ominus z) \succeq A((x \ominus y) \oplus (y \ominus z)) = A(x \ominus y) \,\bar{\wedge}\, A(y \ominus z)$, thus (4) is valid.

Proposition 3.8. *Let A be a cubic ideal of L. If there exists a sequence $\{x_n\}$ of L such that $\lim_{n\to+\infty} A(x_n) = ([1,1],0)$, then $A(0) = ([1,1],0)$.*

Proof. Since A is a cubic ideal of L, then $A(0) \geq A(x)$ for any $x \in L$, it follows that $A(0) \geq A(x_n)$ for any positive integer n. Consider that $([1,1],0) \geq A(0)$, we get $A(0) = ([1,1],0)$.

Let $(L_1, \vee_1, \wedge_1, \oplus_1, \ominus_1, 0_1, 1_1)$ and $(L_2, \vee_2, \wedge_2, \oplus_2, \ominus_2, 0_2, 1_2)$ be two co-residuated lattices. Then $L_1 \times L_2$ is also a co-residuated lattice with respect to the point-wise operations given by:

$$(a,b) \vee (w,v) = (a \vee_1 b, a \vee_2 b),\ (a,b) \wedge (w,v) = (a \wedge_1 b, a \wedge_2 b),$$
$$(a,b) \oplus (w,v) = (a \oplus_1 b, a \oplus_2 b),\ (a,b) \ominus (w,v) = (a \ominus_1 b, a \ominus_2 b),$$

for any $(a,b),(w,v) \in L_1 \times L_2$.

Definition 3.9. *Let $A, B \in \mathbb{C}(L)$. The cartesian product $A \times B$ of A and B is defined by*

$$(A \times B)(x,y) = A(x) \barwedge B(x),$$

for any $(x,y) \in L \times L$. Obviously, $A \times B$ is a cubic set of $L \times L$.

Proposition 3.10. *Let $A, B \in \mathbb{C}(L)$. If A and B are cubic ideals of L, then $A \times B$ is a cubic ideal of $L \times L$.*

Proof. Since A and B are cubic ideals of L, then for any $(x_1,x_2),(y_1,y_2) \in L \times L$, we have
$(A \times B)((x_1,x_2) \oplus (y_1,y_2)) = (A \times B)(x_1 \oplus y_1, x_2 \oplus y_2)$
$= A(x_1 \oplus y_1) \barwedge B(x_2 \oplus y_2)$
$\geq A(x_1) \barwedge A(y_1) \barwedge B(x_2) \barwedge B(y_2)$
$= (A(x_1) \barwedge B(x_2)) \barwedge (A(y_1) \barwedge B(y_2))$
$= (A \times B)(x_1,x_2) \barwedge (A \times B)(y_1,y_2)$.

For any $(x_1,x_2),(y_1,y_2) \in L \times L$, if $(x_1,x_2) \leq (y_1,y_2)$, then $x_1 \leq y_1$ and $x_2 \leq y_2$, and so $A(y_1) \leq A(x_1)$, $B(y_2) \leq B(x_2)$. It follows that $(A \times B)(x_1,x_2) = A(x_1) \barwedge B(x_2) \geq A(y_1) \barwedge B(y_2) = (A \times B)(y_1,y_2)$, hence $A \times B$ is a cubic ideal of $L \times L$.

Proposition 3.11. *Let $A \in \mathbb{C}(L)$. Then A is a cubic ideal of L if and only if $A \times A$ is a cubic ideal of $L \times L$.*

Proof. The sufficiency is very clear by Proposition 3.10, we only need to give the proof of the necessity. We first show that $A(x) \leq A(0)$ for any $x \in L$. In fact that $A(x) = A(x) \barwedge A(x) = (A \times A)(x,x) \leq (A \times A)(0,0) = A(0) \barwedge A(0) = A(0)$, hence $A(x) \leq A(0)$. For $x, y \in L$, it follows that $A(x \oplus y) = A(x \oplus y) \barwedge A(0 \oplus 0) = (A \times A)(x \oplus y, 0 \oplus 0) = (A \times A)((x,0) \oplus (y,0)) \geq (A \times A)(x,0) \barwedge (A \times A)(y,0) = (A(x) \barwedge A(0)) \barwedge (A(y) \barwedge A(0)) = A(x) \barwedge A(y)$, which means that $A(x \oplus y) \geq A(x) \barwedge A(y)$.

For any $x, y \in L$, if $x \leq y$, then $(x,0) \leq (y,0)$, and so $A(x) = A(x) \barwedge A(0) = (A \times A)(x,0) \geq (A \times A)(y,0) = A(y) \barwedge A(0) = A(y)$, that is, $A(x) \geq A(y)$. Hence A is a cubic ideal of L.

In the following, homomorphisms of cubic ideals are defined and some results are studied.

Definition 3.12. *Let f be a mapping from an MV-algebra M_1 into an MV-algebra M_2, and A, B be cubic sets of M_1 and M_2, respectively. Then*

(1) *the preimage $f^{-1}(B)$ of B under f is defined as $f^{-1}(B)(x) = B(f(x))$, for any $x \in M_1$;*

(2) *the image $f(A)$ of A under f is defined as*

$$f(A)(y) = \begin{cases} \overline{\sup}\{A(x) | f(x) = y\}, & f^{-1}(y) \neq \emptyset, \\ ([0,0],1), & otherwise. \end{cases}$$

The following result can be easy proved together with Definition 3.12, and so we omit the proof.

Proposition 3.13. *Let $f : M_1 \to M_2$ be a homomorphism of MV-algebras and A, B be cubic MV-ideals of M_1 and M_2, respectively. Then*

(1) *the preimage $f^{-1}(B)$ is a cubic MV-ideal of M_1;*

(2) *the image $f(A)$ is a cubic MV-ideal of M_2.*

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