Minimum risk estimation of scale parameter of length biased Maxwell distribution with censoring

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ABSTRACT

Length biased distribution arise when the probability of inclusion of population unit in sample is related to the value of the variable measured. For example textile sampling Cox (1969). In this paper the genesis of a length biased Maxwell distribution has been given. The minimum risk estimators of its scale have been obtained under squared error, precautionary and other two loss functions with the help of a type II censored sample. The relative efficiencies have been calculated for the sake of comparison.

Keywords :-Length biased Maxwell distribution (LBMD), loss function , risk function, MMSE estimators, squared error loss function.

1. INTRODUCTION

In many a situation experimenters do not work with truly random sample from the population, in which they are interested, either by design or because of the fact that in many situations it becomes impossible to have random sample from the targeted population. However, since the observations do not have an equal probability of entering the sample, the resulting sampled distribution does not follow the original distribution. Statistical models that incorporate these restriction are called weighted models. The concept of weighted distribution can be traced to Fisher in his paper. "The study of effect of methods of ascertainment upon estimation of frequency" in 1934. Patil, et al. (1986) presented a list of the most common forms of the weighted functions useful in scientific and statistical literature as well as some basic theorems for weighted distribution the length biased sampling was developed by Cox (1962). Gupta and Tripathi (1996) studied the weighted version of the bivariate three parameter logarithmic series distribution. Khatree (1989) presented a useful technique by giving a relationship between the original random variable X and its length biased version Y, when X is either Inverse Gaussian or Gamma distribution. Several authors such as Jain, et al. (1989), Gupta and Kirmani (1990) Sinha,S.K., etc, distribution .,"*Inverse Maxwell Distribution as a survival models, genesis and parameter estimation***"** which is now to be used in the study of the propogation time of Dark Matter ; Singh & Srivastava (2014,a,b,c) studied the classical and Bayesian estimation of size Biased Inverse Maxwell distribution studied the various length-biased distributions and expressed in relation with these of original distributions

Maxwell distribution plays an important role in life testing and reliability theory. The Maxwell distribution is applied in physics & chemistry mainly in Statistical mechanics. Tyagi, R.K. and Bhattacharya, S.K. (1989a,b), considered Maxwell distribution as a life time distribution. They obtained the minimum variance unbiased estimators and Bayes estimators of the parameter and reliability function of the Maxwell distribution. Chaturvedi & Rani(1998) obtained classical and Bayes estimates of reliability function of generalized Maxwell failure distribution, Singh, S.P.(2002) obtained Bayes estimates of parameter and Reliability function of Maxwell distribution under various asymmetric loss functions with censoring using different priors .Bekker & Roux (2005) explored Emperical Bayes estimation for Maxwell distribution. Kazmi et.al.(2012) studied the Bayesian estimation for two component mixture of Maxwell distribution under type-I censoring.

If X is a random variable having the Maxwell distribution, with pdf given by:

$$g(x;\theta) = \frac{4}{\sqrt{\pi}} . \frac{x^2}{\theta^{\frac{3}{2}}} e^{\frac{-x^2}{\theta}} \qquad x > 0, \theta > 0 \tag{1.1}$$

Where θ is scale parameter. the rth row moments are given by

$$\mu_r' = \frac{2}{\sqrt{\pi}} \theta^{\frac{r}{2}} \Gamma\left(\frac{r+3}{2}\right), \tag{1.2}$$

The mean and variance are obtained as

$$\mu_1' = 2\sqrt{\frac{\theta}{\pi}} ; \tag{1.3}$$

And

$$\mu_2 = \frac{\theta(3\pi-8)}{2\pi}; \quad (1.4)$$

In this paper the estimation of the parameter of the length biased Maxwell distribution is considered .Let T be a random variable having the length biased Maxwell distribution, its pdf comes out to be

$$f(t;\theta) = \frac{2}{\theta^2} t^3 e^{\frac{-t^2}{\theta}}; \ t,\theta>0 \quad (1.5)$$

using the relationship

$$f(t;\theta) = \frac{tg(t,\theta)}{E(t)}$$

where $g(t;\theta)$ follows Maxwell distribution as given in (1.1)

Let us suppose that n items are put to test for their life times and the experiment is terminated when r(< n) items have failed, If $t_1,\ldots,t_n$. denote the first r observations having common pdf as given in (1.5), then the joint pdf is given by:-

$$f(t;\theta) = \frac{\Gamma n}{\Gamma(n-r)} \frac{2^r}{\theta^{2r}} \prod_{i=1}^{r} t_i^3 \ e^{\frac{-x^2}{\theta}} \ t,\theta>0; \quad (1.7)$$

now if $z = [\sum_{i=i}^{r} t_i^2 + (n-r)\, t_r^2];$ (1.8)

the maximum likelihood estimator (MLE) $\hat{\theta}$ of θ may be obtained as;

$$\hat{\theta} = \frac{z}{2r}; \quad (1.9)$$

The pdf of $\hat{\theta}$ is given by

$$f(\hat{\theta}) = \frac{1}{\Gamma(2r)} \left(\frac{2r}{\theta}\right)^{2r} \hat{\theta}^{2r-1} e^{\frac{-2r\hat{\theta}}{\theta}} \hat{\theta}>0 \quad (1.10)$$

2 THE MINIMUM EXPECTED LOSS (RISK) ESTIMATOR UNDER SQUARED ERROR LOS FUNCTION

The Expected loss (risk) in this case is the Mean Squared Error (MSE) now define

$$\text{MSE}(\hat{\theta}) = \int_0^\infty (\hat{\theta}-\theta)^2 f(\hat{\theta})d\hat{\theta}$$

$$= \int_0^\infty [\hat{\theta}^2 - 2\theta\hat{\theta} + \theta^2] f(\hat{\theta})d\hat{\theta} \quad (2.1)$$

Putting the value of $f(\hat{\theta})$ from (1.10) we get

$$\text{MSE}(\hat{\theta}) = \int_0^\infty [\hat{\theta}^2 - 2\theta\hat{\theta} + \theta^2] \frac{1}{\Gamma 2r} \left\{\frac{2r}{\theta}\right\}^{2r} \hat{\theta}^{2r-1} \boldsymbol{exp}\left\{\frac{2r\hat{\theta}}{\theta}\right\} \mathbf{d}\hat{\theta} \quad (2.2)$$

$$\text{MSE}(\hat{\theta}) = \frac{\theta^2}{2r} \quad (2.3)$$

MINIMUM MEAN SQUARE ERROR ESTIMATOR

Let us define

$$\theta^* = M\hat{\theta} \quad (2.4)$$

Now

$$\text{MSE}(\theta^*) = E_\theta(\theta^* - \theta)^2 = E_\theta(M\hat{\theta} - \theta)^2$$

$$= M^2 E_\theta(\hat{\theta}^2) - 2\theta M E_\theta(\hat{\theta}) + \theta^2 \quad (2.5)$$

$$\frac{d\text{MSE}(\theta*)}{dM} = 2\, M E_\theta(\hat{\theta}^2) - 2\theta\, E_\theta(\hat{\theta})$$

$$\frac{d^2\text{MSE}(\theta*)}{dM^2} = 2E(\hat{\theta}^2) \geq 0$$

and

$\frac{d\text{MSE}(\theta*)}{dM} = 0$ leads to absolute minimum of MSE (θ*)

Now $M = \frac{2r}{2r+1}$ (2.6)

Here we see that M is independent of θ

Now $\theta^* = \frac{z}{2r+1}$ (2.7)

is MMSEE of θ

The relative efficiencies of the estimator θ* with respect to $\hat{\theta}$ is defined as

$$\text{Rel.eff.}(\theta^* / \hat{\theta}) = \frac{\text{MSE}(\hat{\theta})}{\text{MSE}(\theta^*)} = 1+\frac{1}{2r} > 1; \quad \text{for} \quad r \geq 1 \tag{2.8}$$

3.THE MINIMUM EXPECETED LOSS (RISK) ESTIMATORS PRECAUTIONARY LOSS FUNCTION

This loss function was introduced by Norstrom(1996) ,is a very useful and simple precautionary loss function is given as

$$L(\Delta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \tag{3.1}$$

The risk function of MLE $\hat{\theta}$ under precautionary loss functio ,denoted by $R_P(\hat{\theta})$ is defined as

$$R_P(\hat{\theta}) = E(L(\Delta)) = \int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} f(\hat{\theta}) d\hat{\theta} \tag{3.2}$$

On using (1.10) we get

$$R_P(\hat{\theta}) = \int_0^\infty \left(\hat{\theta} + \frac{\theta^2}{\hat{\theta}} - 2\theta\right) \frac{1}{\Gamma 2r} \left\{\frac{2r}{\theta}\right\}^{2r} \hat{\theta}^{2r-1} \boldsymbol{exp}\left\{\frac{2r\hat{\theta}}{\theta}\right\} \mathbf{d}\hat{\theta}$$

$$R_P(\hat{\theta}) = \left[\frac{\theta}{(2r-1)}\right] \tag{3.3}$$

Similarly the risk function under precautionary loss function for θ* is given by, (where $\theta^* = M\hat{\theta}$ and $\Delta = (\theta^* - \theta)$)

$$R_P(\theta^*) = \int_0^\infty \frac{(\theta^*-\theta)^2}{\theta^*} f(\hat{\theta}) d\hat{\theta} = \int_0^\infty \left(\theta^* + \frac{\theta^2}{\theta^*} - 2\theta\right) f(\hat{\theta}) d\hat{\theta}$$

$$= \frac{\mathbf{1}}{\boldsymbol{\Gamma 2r}} \int_0^\infty \left(M\hat{\theta} + \frac{\theta^2}{M\hat{\theta}} - 2\theta\right) \left\{\frac{2r}{\theta}\right\}^{2r} \hat{\theta}^{2r-1} \boldsymbol{exp}\left\{\frac{-2r\hat{\theta}}{\theta}\right\} \mathbf{d}\hat{\theta}$$

$$R_P(\theta^*) = \theta\left[M + \frac{2r}{M(2r-1)} - 2\right] \tag{3.4}$$

In order to get the value of M that minimizes $R_P(\theta^*)$ we proceed as follows-

$$\frac{d}{dM} R_P(\theta^*) = \theta\left[M + \frac{2r}{M(2r-1)} - 2\right] = 0$$

$$\text{Where } M = \left(\frac{2r}{2r-1}\right)^{\frac{1}{2}} \tag{3.5}$$

Now $\frac{d^2\text{MSE}(\theta^*)}{dM^2} = \frac{4r\theta}{(2r-1)M^3} > 0$

Thus the value of M obtained in (3.5) will lead to absolute minimum of $R_P(\theta^*)$ .here we see that M doesn't depend on any unknown parameter so its exact value is determined.

Define the relative efficiency as

$$\text{Rel.eff.}(\theta^* / \hat{\theta}) = \frac{\text{MSE}(\hat{\theta})}{\text{MSE}(\theta^*)} = \frac{\left[\frac{1}{(2r-1)}\right]}{\left[M + \frac{2r}{M(2r-1)} - 2\right]} \tag{3.6}$$

$$\frac{1}{(2r-1)} \geq 2\left(\sqrt{\frac{2r}{2r-1}} - 1\right) \tag{3.7}$$

$r \geq \frac{1}{2}$ which is always true.

In order to have the $\text{Rel.eff.}\left(\frac{\theta^*}{\hat{\theta}}\right) > 1$

4 THE MINIMUM EXPECTED LOSS (RISK) ESTIMATOR UNDER OTHER LOSS FUNCTION (I)

A useful Loss function is given by

$$L(\Delta) = \left(\frac{\hat{\theta}}{\theta} - 1\right)^2 \tag{4.1}$$

Risk function under $L(\Delta)$ of MLE $\hat{\theta}$ is denoted by $R_A(\hat{\theta})$ as

$R_A(\hat{\theta})$ =E(L(Δ)) $=\int_0^\infty \left(\frac{\hat{\theta}}{\theta}-1\right)^2 f(\hat{\theta})d\hat{\theta}$

$R_A(\hat{\theta}) = \int_0^\infty \left(\frac{\hat{\theta}^2}{\theta^2}-\frac{2\hat{\theta}}{\theta}+1\right)\frac{1}{\Gamma 2r}\left\{\frac{2r}{\theta}\right\}^{2r}\hat{\theta}^{2r-1}\exp\left\{\frac{2r\hat{\theta}}{\theta}\right\}d\hat{\theta}$

$R_A(\hat{\theta}) = [\frac{1}{2r}]$ (4.2)

Let us define $\theta^*=M\hat{\theta}$ (4.3)

$R_A(\theta^*) =\int_0^\infty \left(\frac{\hat{\theta}}{\theta}-1\right)^2 f(\hat{\theta})d\hat{\theta} =\int_0^\infty \left(\frac{\hat{\theta}^2}{\theta^2}-\frac{2\hat{\theta}}{\theta}+1\right) f(\hat{\theta})d\hat{\theta}$

$=\frac{1}{\Gamma 2r}\int_0^\infty \left(\frac{M\hat{\theta}^2}{\theta^2}-\frac{2M\hat{\theta}}{\theta}+1\right)\left\{\frac{2r}{\theta}\right\}^{2r}\hat{\theta}^{2r-1}\exp\left\{\frac{2r\hat{\theta}}{\theta}\right\}d\hat{\theta}$

$R_A(\theta^*) = [\frac{M^2(2r+1)}{2r}-2M+1]$ (4.4)

Now in order to M lead to minimum of $R_A(\theta^*)$ .we must have,

$$\frac{dR_A(\theta^*)}{dM}=\frac{2M(2r+1)}{2r}-2=0$$

Which leads to the value of M as

$M=\frac{2r}{2r+1}$ (4.5)

We see that M does not depend on any known parameter so its exact value may be obtained and it leads to absolute minimum of $R_A(\theta^*)$ since

$\frac{d^2R_A(\theta^*)}{dM^2} = \frac{(2r+1)}{2r} > 0$ (4.6)

Define the relative efficiency of θ^* with respect to $\hat{\theta}$ as

Rel.eff.$(\theta^* / \hat{\theta}) = \frac{MSE(\hat{\theta})}{MSE(\theta^*)} = \frac{\frac{1}{2r}}{\frac{M^2(2r+1)}{2r}-2M+1}$ (4.7)

$\frac{1}{2r} > \frac{1}{2r+1}$ (4.8)

In order to have the Rel.eff.$\left(\frac{\theta^*}{\hat{\theta}}\right) > 1$

5. THE MINIMUM EXPECTED LOSS(RISK) ESTIMATOR UNDER OTHER LOSS FUNCTION (I)

A useful Loss function is given by

$L(\Delta) =\left(\frac{\theta}{\hat{\theta}}-1\right)^2$ (5.1)

Risk function under $L(\Delta)$ of MLE $\hat{\theta}$ is denoted by $R_B(\hat{\theta})$ as

$R_B(\hat{\theta})$ =E(L(Δ)) $=\int_0^\infty \left(\frac{\theta}{\hat{\theta}}-1\right)^2 f(\hat{\theta})d\hat{\theta}$

$R_B(\hat{\theta}) = \int_0^\infty \left(\frac{\theta^2}{\hat{\theta}^2}-\frac{2\theta}{\hat{\theta}}+1\right)\frac{1}{\Gamma 2r}\left\{\frac{2r}{\theta}\right\}^{2r}\hat{\theta}^{2r-1}\exp\left\{\frac{2r\hat{\theta}}{\theta}\right\}d\hat{\theta}$

$R_B(\hat{\theta}) =\left[\frac{(2r)^2}{(2r-1)(2r-2)}-\frac{4r}{(2r-1)}+1\right]$ (5.2)

Let us define $\theta^* = M\hat{\theta}$ (5.3)

$$R_B(\theta^*) = \int_0^\infty \left(\frac{\theta}{\theta^*} - 1\right)^2 f(\hat{\theta})d\hat{\theta} = \int_0^\infty \left(\frac{\theta^2}{\theta^{*2}} - \frac{2\theta}{\theta^*} + 1\right) f(\hat{\theta})d\hat{\theta}$$

$$= \frac{1}{\Gamma 2r}\int_0^\infty \left(\frac{\theta^2}{(M\,\hat{\theta})^2} - \frac{2\theta}{M\hat{\theta}} + 1\right)\left\{\frac{2r}{\theta}\right\}^{2r} \hat{\theta}^{2r-1} \exp\left\{\frac{2r\hat{\theta}}{\theta}\right\} d\hat{\theta}$$

$$R_B(\theta^*) = \left[\frac{(2r)^2}{(2r-1)(2r-2)M^2} - \frac{4r}{M(2r-1)} + 1\right] \quad (5.4)$$

Now in order to M lead to minimum of $R_B(\theta^*)$ .we must have,

$$\frac{dR_B(\theta^*)}{dM} = -\frac{2(2r)^2}{(2r-1)(2r-2)M^3} + \frac{4r}{(2r-1)M^2} = 0$$

Which leads to the value of M as

$$M = \frac{r}{(r-1)},\ r>1 \quad (5.5)$$

We see that M does not depend on any known parameter so its exact value may be obtained and it leads to absolute minimum of $R_B(\theta^*)$ since

$$\frac{d^2R_B(\theta^*)}{dM^2} = \frac{6(2r)^2}{(2r-1)(2r-2)M^4} + \frac{8r}{(2r-1)M^3} > 0$$

$$\frac{d^2R_B(\theta^*)}{dM^2} = \frac{4(r-1)^3}{(2r-1)r^2} > 0 \quad \text{for} \quad r>1 \quad (5.6)$$

Define the relative efficiency of θ^* with respect to $\hat{\theta}$ as

$$\text{Rel.eff}\,\theta^*/\hat{\theta}) = \frac{\text{MSE}(\hat{\theta})}{\text{MSE}(\theta^*)} = \frac{\left[\frac{(2r)^2}{(2r-1)(2r-2)} - \frac{4r}{(2r-1)} + 1\right]}{\left[\frac{(2r)^2}{(2r-1)(2r-2)M^2} - \frac{4r}{M(2r-1)} + 1\right]}$$

$$\text{Rel.eff.}(\theta^* / \hat{\theta}) = \frac{r+1}{r-1} > 1 \quad (5.7)$$

In order to have the Rel.eff.$(\theta^* / \hat{\theta}) > 1$

TABLE

	Precautionary loss function		Loss function-1		Loss function-2	
R	M	Rei.eff.	M_1	Rei.eff.	M_2	Rei.eff.
5	1.054093	1.027046	0.909091	1.100000	1.250000	1.500000
10	1.025978	1.012989	0.952381	1.050000	1.111111	1.222222
15	1.017095	1.008548	0.967742	1.033333	1.071429	1.142857
20	1.012739	1.006370	0.975610	1.025000	1.052632	1.105263
25	1.010153	1.005076	0.980392	1.020000	1.041667	1.083333
30	1.008439	1.004220	0.983607	1.016667	1.034483	1.068965

35	1.007220	1.003610	0.985915	1.014286	1.029412	1.058824
40	1.006309	1.003155	0.987654	1.012500	1.025641	1.051282
45	1.005602	1.002801	0.989011	1.011111	1.022727	1.045455
50	1.005038	1.002519	0.990099	1.010000	1.020408	1.040816
55	1.004577	1.002288	0.990991	1.009091	1.018519	1.037037
60	1.004193	1.002097	0.991736	1.008333	1.016949	1.033898
65	1.003868	1.001934	0.992366	1.007692	1.015625	1.031250
70	1.003591	1.001795	0.992908	1.007143	1.014493	1.028986
75	1.003350	1.001675	0.993378	1.006667	1.013514	1.027027
80	1.003140	1.001570	0.993789	1.006250	1.012658	1.025316
85	1.002954	1.001477	0.994152	1.005882	1.011905	1.023810
90	1.002789	1.001395	0.994475	1.005556	1.011236	1.022472
95	1.002642	1.001321	0.994764	1.005263	1.010638	1.021411
100	1.002509	1.001255	0.995025	1.005000	1.010101	1.020320

Conclusion

Thus θ^* is most efficient in its class for all values of r when precautionary loss function is the criterion of selection of the estimator.

In tables 1 and 2 the exact values of M (since M do not depend on any unknown parameter) have been tabulated along with the relative efficiencies of the MELO estimators θ^* with respect to MLE $\hat{\theta}$, under precautionary and other two asymmetric loss functions. These tables show that MELO estimator θ^* is preferable to $\hat{\theta}$ for all practical purposes.. These MELO estimators θ^* uniformly dominates the MLE $\hat{\theta}$ as the Rel. eff.($\theta^* / \hat{\theta}$) > 1 (inequalities (2.8), (3.7), (4.8), and are always true).

The estimator θ^* defined is (2.7) is MMSE estimator which always dominates $\hat{\theta}$.

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