

# OPTIMISATION OF PULSE FORMING NETWORKS

Ayush Kale<sup>1</sup>

<sup>1</sup>*Department of Electrical Engineering, College of Engineering, Shiv Nadar University, NH91, Tehsil Dadri, Greater Noida, Uttar Pradesh (201314), India*

<sup>1</sup>ak788@snu.edu.in

**Abstract-**This article presents reliable optimization techniques to improve the pulse generated by the Pulse Forming Networks (PFN) for high energy efficiency. The shape generated by PFN is not ideal however PFNs can be designed with better pulse performance by doing modifications and using techniques which can improve the desired output pulse shape and overall energy efficiency of the network. In this study, two novel techniques have been discussed. Particle Swarm Optimization (PSO) algorithm, proposed by Kennedy & Eberhart, a metaheuristic algorithm has been used to calculate the exact values of PFN with variable inductors which would tune PFN to generate the pulses with high energy efficiency. Green's Function approach developed by Carl Neumann has been used to calculate the compensating voltage which can be used to compensate for the ripples, overshoot, rise time, fall time, etc. to improve the overall pulse performance. Using these optimization techniques, the energy efficiency of the network can be improved to a great extent. Both the techniques are validated using MATLAB/Simulink models.

**Keywords:** -Pulse Forming Networks (PFN), Voltage Correction, Pulse Performance, Fall time, Overshoot, Particle Swarm Optimisation (PSO), Green's Function.

## I. INTRODUCTION

Pulse power technology is an important field since it has a variety of applications almost in every sphere of modern technology, right from high energy physics to sewage treatment. Pulse power technology deals with high power pulse generators or pulse modulators which generate high peak power short pulses repeated at some rep rate. Based on the type of discharge switches, there are many topologies like- Line type modulators with Thyatron switch, Modulators with single solid-state high voltage switch and a capacitor bank, Marx generator with solid-state switches, Pulse Modulators with matrix transformer etc.

Among them, pulse modulators employing Pulse Forming Network (PFN), also popularly Line type modulators, are still popular because of their less complex, robust design with high-efficiency performance. PFN is basically a cascaded network of low-pass filters with many inductors and capacitors. They store exactly the amount of energy required to generate a single pulse discharge this energy into the load in the form of a pulse of a specified shape. Therefore, pulse generators with PFNs do not need crowbar protection for the expensive loads they drive, because in an event of arcing in the load the amount of energy available is minimum compared to other topologies.

However, one of the concerns of PFN based pulse generator is the shape of the pulse it generates. The pulse shape not perfectly rectangular as in the case of pulse generators with solid-state discharge switches with a capacitor bank. Pulse shapes become worst when the values of high voltage capacitors employed for fabrication differ from the designed values. With aging values of high voltage capacitors change which also causes a change in pulse shape over the period of continuous operation. Pulses nearing rectangular are the best as they have the highest energy efficiency, which measures the overall performance of the pulse. It is nothing but the ratio of energy stored in the flat-top to the total energy of the pulse. To cater to this problem, PFNs are made with either employing variable tunable inductors or an additional compensating network to compensate the ripples on the flat top, droop, overshoot, etc. to improve their energy efficiency.

In this article, two optimization techniques that employ Particle Swarm Optimization (PSO) and green's functions are presented, which can be used to modify PFNs such that they generate pulses with better pulse performance.

Section-2 defines the standard pulse shape definitions and related terms and in section-3 standard design principles are described to design Pulse Forming Network (PFN). Using standard formulae given in section-3, some PFNs are designed and modeled in Simulink to explain both the optimization techniques given in section-4 and section-5 followed by a conclusion.

## II. PULSE SHAPE DEFINATION AND RELATED TERM

Pulse Performance is measured in terms of rise-time, fall-time, overshoot, flat top width, ripple content on the flat top, and under-shoot. Fig.-1 shows a pulse showing all the standard parameters, which define the performance of the pulse. [1]

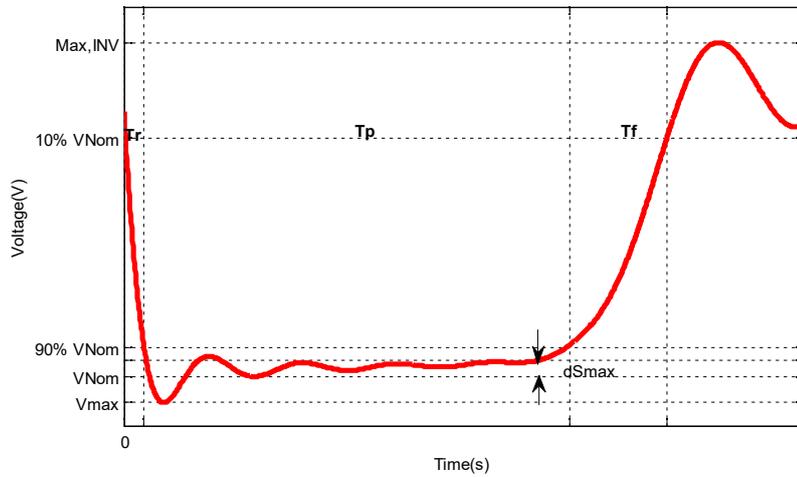


Fig. 1 Pulse Shape definitions and related terms

Rise time ( $T_r$ ) and Fall time ( $T_f$ ) are normally specified between 10% and 90% of the average pulse amplitude. Flat top ( $T_p$ ) is the width available between 90% of nominal voltage ( $V_n$ ). Overshoot is the maximum voltage ( $V_{max}$ ) that shoots above the nominal voltage level, whereas undershoot or inverse voltage ( $V_{max, inv}$ ) appears at the trailing edge of the pulse. Droop ( $S_{max}$ ) is a term that defines a fall in voltage level towards the trailing edge of the pulse. Ripple on the flat top (% R) is peak-to-peak voltage above and below the nominal voltage in the flat top region. Overall performance can be measured in terms of energy efficiency which is defined as (I).

$$\eta = \frac{W_{top}}{T_b \int_0^{T_b} v(t) \times i(t) dt} \quad \text{---(I)}$$

Where  $W_{top}$  is energy of the pulse in flat top region of the pulse;  $T_b$  is the pulse duration at the bottom (i.e.,  $T_b = T_r + T_p + T_f$ ), and  $v(t)$  and  $i(t)$  are instantaneous values of voltage and current during the pulse. The pulse performance is dictated by pulse generator design as well as on pulse transformer if it is employed with the pulse generator[2].

## III PULSE FORMING NETWORK

Pulse Forming Network (PFN) resembles a Transmission Line with lumped finite parameters. Basically, the network is cascaded low pass filters with Inductors and Capacitors with several sections. The values of Capacitors and Inductors depends upon the number of sections, the Characteristic Impedance of the network, and the pulse width which it generates when discharged on the load equivalent to its characteristic impedance. Pulse width, characteristic impedance, number of turns are related to the inductance and capacitance of each cell by the following formulae.[2]

$$Z_0 = \sqrt{\frac{L}{C}} \quad \Omega \quad \text{---(II)}$$

$$\tau = 2n\sqrt{LC} \quad \text{sec} \quad \text{---(III)}$$

Solving (II) and (III) results in Inductance and Capacitance of values L and C given as

$$L = \frac{\tau \times Z_0}{2n} \quad H$$

---(IV)

$$C = \frac{\tau}{2n \times Z_0} F \quad \text{---(V)}$$

L and C values are given for one-second pulse width one Ohm impedance for n section network are given as  $L = \frac{1}{2n} H$  &

$C = \frac{1}{2n} F$ . With this, inductance and capacitance for any pulse width ( $\tau$ ) and characteristic impedance ( $Z_0$ ) can be

calculated by multiplying inductance by  $\tau \times Z_0$  and Capacitance by,  $\frac{\tau}{Z_0}$ .

#### IV OPTIMIZATION USING PSO METHOD

PFNs resemble a transmission line with a finite number of cells and due to which the pulses generated by them are inherent with ripples on flat top even if the cell inductances and capacitances are perfect. It is impossible to generate a rectangular pulse by means of a lumped parameter network. [ Glasso]. The ripples further increase if the capacitance of high have some tolerance band and change with aging. Typically, high-power pulse modulators are characterized by short pulses of the order of maximum tens of microseconds. They employ variable air-core inductors which are changed either by shorting their turns or inserting conducting slug normally made from copper.

Since typically pulse width generated by Pulse Forming Networks are of the order of a few micro-seconds, they are employed with tunable air-core inductors. The design of an air-core inductor is done by employing the following empirical formula[3]:

$$L_N = \frac{r^2 \times n^2}{9r + 10l} (uH) \quad \text{--- (VI)}$$

where r is the radius of coil, n the number of turns and l its length in inches. The formula is accurate within 1% for  $l > 0.8 r$ . [2] Tuning can be done by employing a conducting shield that confines the magnetic field from linking with turns, thereby reducing the inductance of the coil or by simply shorting the turns. The effect of shield, including the end effect, can be defined by the following equations[3].

$$L_a = L_o \times \left[ 1 - \left( \frac{rs}{rc} \right)^2 \times \left( \frac{ls}{lc} \right) \times \frac{1}{k} \right] \quad \text{---(VII)}$$

Where  $L_a$  is the actual inductance of the coil,  $L_o$  the inductance of coil without tuner,  $l_s$  and  $l_c$  is the length of shield and length of the coil, and  $K$  is a factor which a function of diameter coil to the length of the coil.[2]. Decrease in the inductor is associated with an increase in effective resistance of the coil. Since inductance is decreased by the induced current flowing circumstantially on the surface of the shield, such currents heat the shield dissipating power.

Ripple frequency on the flat top depends upon the number of cells in the PFN. An increasing number of cells results in small ripples on the flat top. Furthermore, employing a large number of cells and tuning them properly ensures uniform convergence resulting in small overshoot, and oscillations on flat top can be reduced.[1] However, an increasing the number of cells with the same fixed cell inductors and capacitors results in a fast rise time with a large overshoot at the beginning. With a large number of cells values of inductance per cell decreases for given characteristic impedance which makes it difficult to design tunable inductors. Instead of having PFN with a large number of cells, multiple PFNs can be connected in parallel which results in higher impedance and results in larger values inductance per cell which provides greater flexibility in designing tunable inductors. These PFNs can be tuned in such a way that the ripples generated by individual PFNs fall in opposite phases

and cancel out and generates a pulse with a better flat top. Tuning PFN inductors for a large number of cells manually cumbersome.

Particle Swarm Optimization (PSO) algorithm, proposed by Kennedy & Eberhart, is a metaheuristic algorithm that is appropriate to optimize nonlinear continuous functions. The algorithm is based on the social behavior of animals like birds.[4] In a group of fish searching for food, each bird has a position and velocity within the search space. They change their position by adjusting velocity based on the individual's flying experience as well as the experience of the group as a whole by memorizing their best position and sharing this with the group members. PSO is the most popular algorithm for solving minimization and maximization problems due to its simplicity.

The goal of an optimization problem is to determine a variable represented by a vector,  $X = [x_1, x_2, x_3, \dots, x_n]$  that minimizes or maximizes depending on the proposed optimization formulation of the function  $f(X)$ . Function  $f(x)$  is called an objective function which is one of the most important parts of the algorithm.

There are four steps involved in solving the problem using PSO.

**Step-1:** Initialize particles and velocity randomly within search space.

**Step-2:** Consider a group of P particles, which has a position vector,  $X_i^t = [x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}]^T$  and velocity vector,  $V_i^t = [v_{i1}, v_{i2}, v_{i3}, \dots, v_{in}]^T$  at iteration for each  $i^{\text{th}}$  particle that it composes it. These vectors are updated through the dimension j according to the following equations.

$$V_{ij}^{t+1} = wV_{ij}^t + c1.r1^t.(Pbest_{ij} - X_{ij}^t) + c2.r2^t.(Gbest_j - X_{ij}^t) \text{---(VIII)}$$

$$X_{ij}^{t+1} = X_{ij}^t + V_{ij}^{t+1} \text{---(IX)}$$

w is the inertia weight; c1 and c2 are the acceleration constants; r1t and r2t are random numbers between 0 to 1;  $P_{best}$  &  $G_{best}$  are the individuals and global best positions. First-term in the velocity is called inertia component, the second term as a cognitive component, and the third term as social component.

**Step-3:** Evaluate the objective function  $f(x)$  and update the population irrespective of its fitness value.

**Step-4:** Step-1 to Step-3 is repeated until the stopping criterion is met.

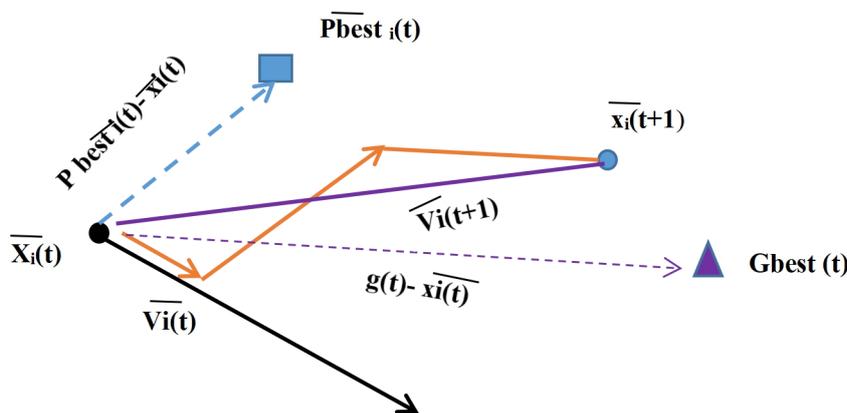


Fig.2 Movement of particles in PSO

Fig.2 shows the movement of particles in the PSO method.  $\mathbf{x}_i(t)$  is particle position for iteration 't' and  $x_i(t+1)$  is the position calculated for next iteration (t+1).  $V_i(t)$  is the old velocity and  $V_i(t+1)$  is the new calculated velocity.  $P_{best}i(t)$  is personal best from previous and  $G_{best}(t)$  is global best.

Output pulse voltage from the PFN is a function of cell inductances, cell capacitances, charging voltage, and load impedance. The values of Inductors are the only variables since all other parameters except are fixed. Particle Swarm Optimization (PSO) algorithm is used for determining a set of inductor values represented by  $L = [L_1, L_2, L_3 \dots L_n]$  which are optimized to meet the goal of the objective function. Inductors' values are set within the upper and lower limit which are practically realization. Here, the objective function is a MATLAB function that runs a Simulink model of Pulse Forming Network that takes all its parameters along with target voltage level from the main function implementing PSO algorithm and calculates the root mean square error between the target value and simulated values of the pulse over the selected part of the flat top.

The main function implementing the PSO algorithm does Step-1 to Step-4 and determines the optimum values of Inductors which minimizes the error to a minimum, and with these parameters, PFN generates a pulse with a good flat top.

The constriction Factor Approach proposed by Clerc and Kennedy in 2002 is used for defining inertia weight,  $w$ , and acceleration coefficients  $c_1$  and  $c_2$  to ensure convergence. As per this approach  $c_1$ ,  $c_2$  and  $w$  are defined as follows [5]: -

$$\chi = \frac{2\kappa}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|} \text{---(X)}$$

Where  $\phi = \phi_1 + \phi_2 \geq 4$  and  $0 \leq \kappa \leq 1$  and  $c_1 = \chi\phi_1$ ,  $c_2 = \chi\phi_2$ ,  $w = \chi$

From (X) PSO parameters  $c_1$ ,  $c_2$  and  $w$  are calculated as  $c_1=1.4962, c_2=1.4962$  and  $w=0.7298$  considering  $\kappa = 1$  and

$\phi_1 = \phi_2 = 2.05$ .

Initially, a Pulse Forming Network of 8 cells is considered, which produces a 1-sec pulse with nominal values of inductance and capacitance calculated using the formulae in section -3. A Simulink model for single PFN with 8 cells which is a part of the objective function is shown in Fig. 3. The main PSO MATLAB function shows the waveforms before and after optimization. Fig. 4, and Fig. 5 show PFN output waveforms, namely before and after optimization, cell inductances, before and after optimization, and cell capacitances[6].

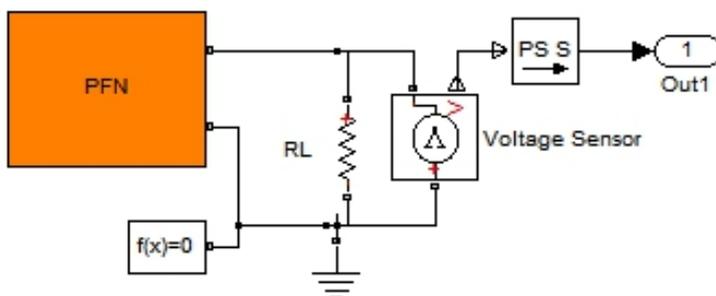


Fig. 3. Simulink Model for Single PFN with 8 cells

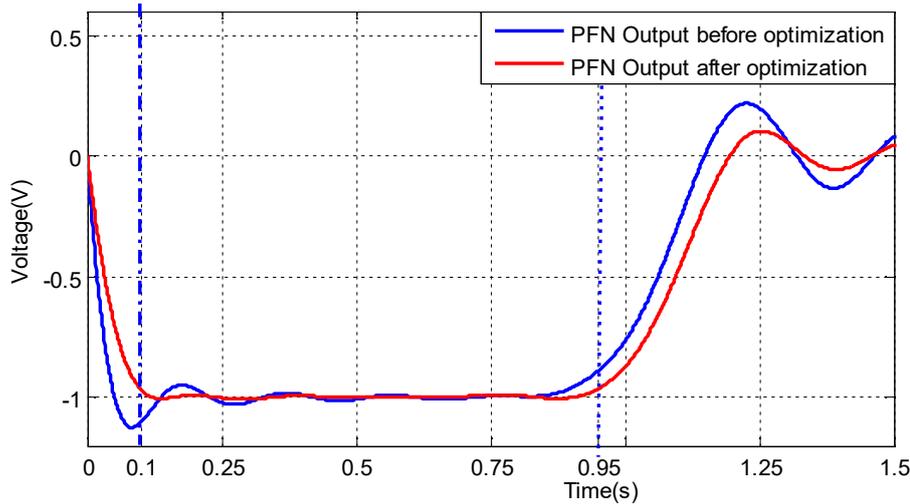


Fig. 4. Output waveforms, before and after optimization for configuration with single PFN.

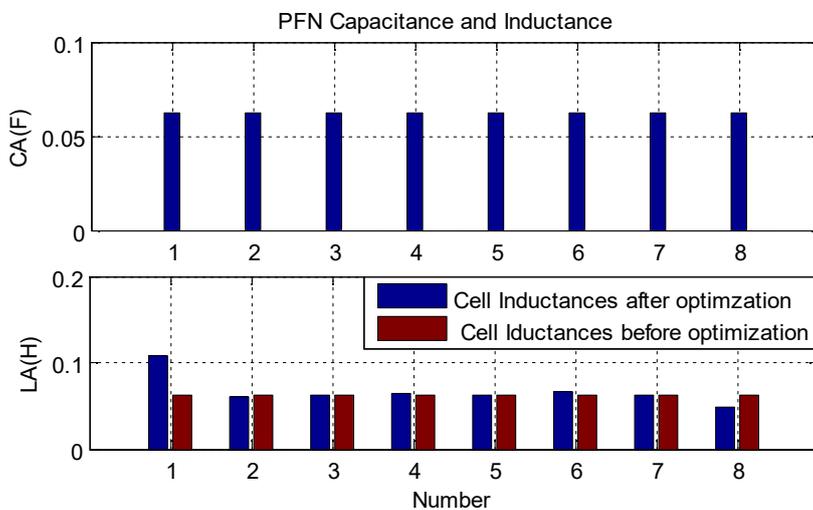


Fig. 5. Cell inductances and capacitances before and after optimization for configuration with single PFN.

A parametric study is done to show that if two PFNs or more if connected in parallel, the configuration produces a pulse with a better flat top even with +/- 5% error in capacitors from nominal values. Currents contributed by individual PFNs take shapes in such a way that they cancel each other to generate a pulse with better performance. A Simulink model for two PFN with 8 cells which is a part of the objective function is shown in Fig. 6. Nominal values of inductance and capacitance calculated using the formulae in section -3 for a 1-sec pulse.

Fig. 7, show PFN output waveforms, namely before and after optimization, and Fig. 8 shows cell inductances and capacitances, before and after optimization. It is evident from Fig. 9 that the ripples on current from PFN-1 and PFN-2 cancel each other after optimization[7].

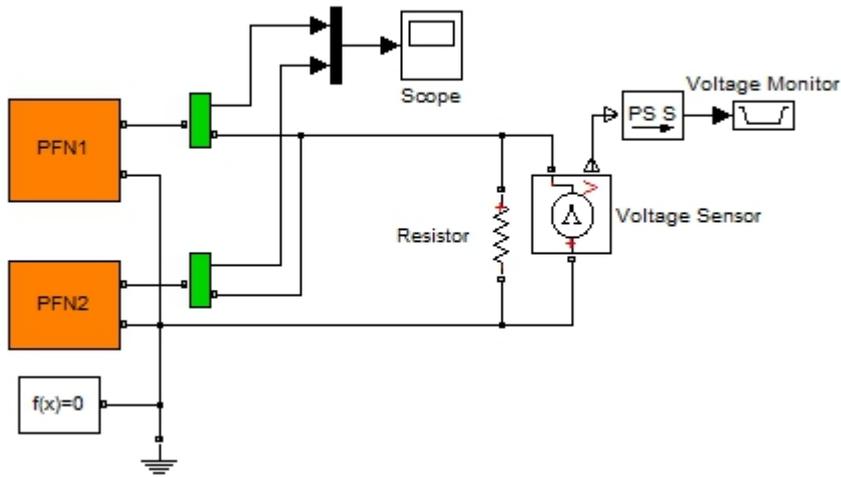


Fig. 6 Simulink Model for 2 PFN Models in Parallel

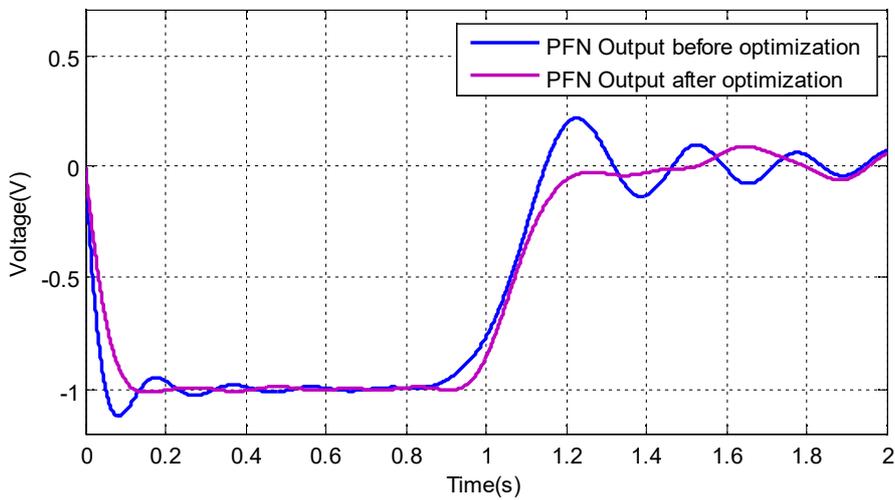


Fig. 7 Output waveforms before and after optimization for configuration with two PFNs in parallel

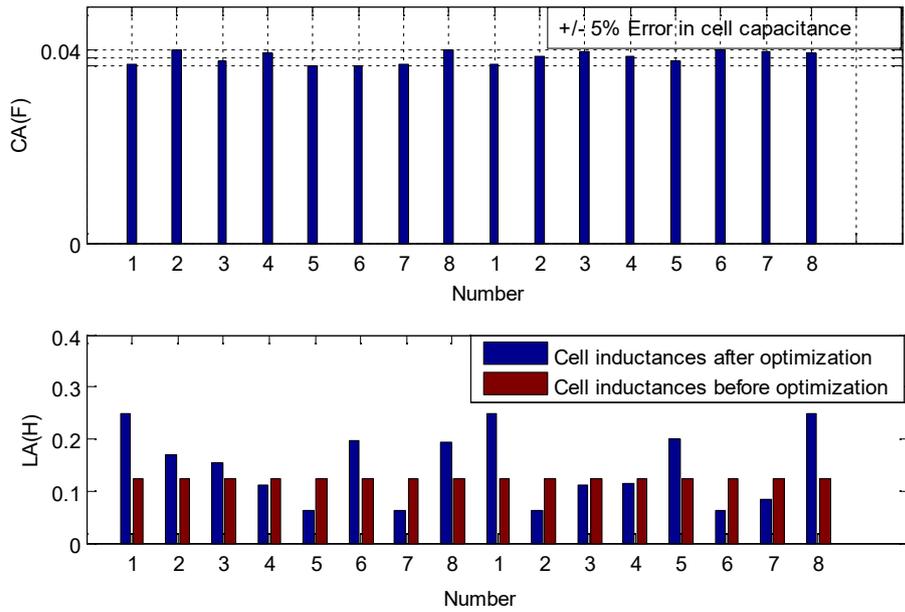


Fig. 8 Cell inductances and capacitances before and after optimization for configuration with two PFNs in parallel.

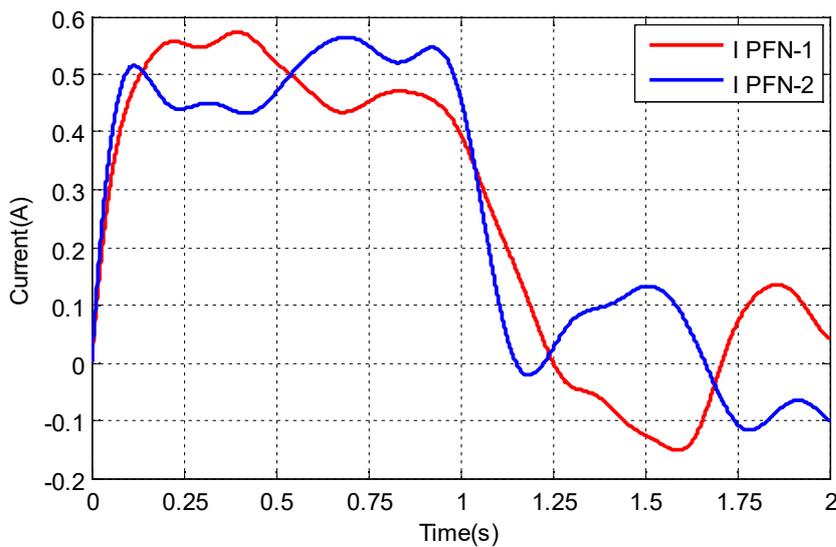


Fig. 9 Output currents from PFN-1 and PFN-2 after optimization for configuration with two PFNs in parallel.

PFNs are often used with pulse transformers for impedance matching with the load for maximum power transfer while stepping up the voltage to desired level. Output pulse with pulse transformer is always associated with some droop due swing in flux density towards saturation due increase of magnetizing current during the pulse width of the pulse. However, this droop can be reduced by designing the pulse transformers with higher magnetizing inductor but at cost of slow rise and fall time. Since several percentage of droop can be compensated by optimizing PFN inductors, it allows the designer to design the pulse transformer with with fast rise and fall times, allowing some droop. Therefore, using optimization of

inductor values results in output pulse with high energy efficiency even with pulse transformer. In the next parametric study done below, 5% droop is intentionally introduced in addition to +/- 5% error in PFN capacitors, and variable inductors are optimized in similar way using PSO. In Fig. 10 shows the PFN output before and after optimization along with cell inductances and capacitance in Fig. 11.

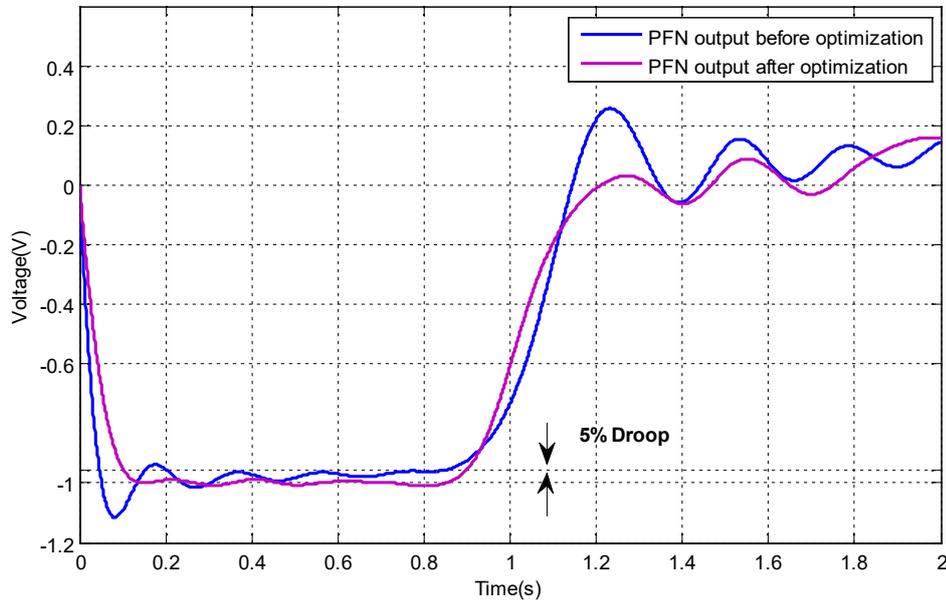


Fig.-10 Output waveform before and after optimization for configuration of two PFNs in parallel for compensating droop.

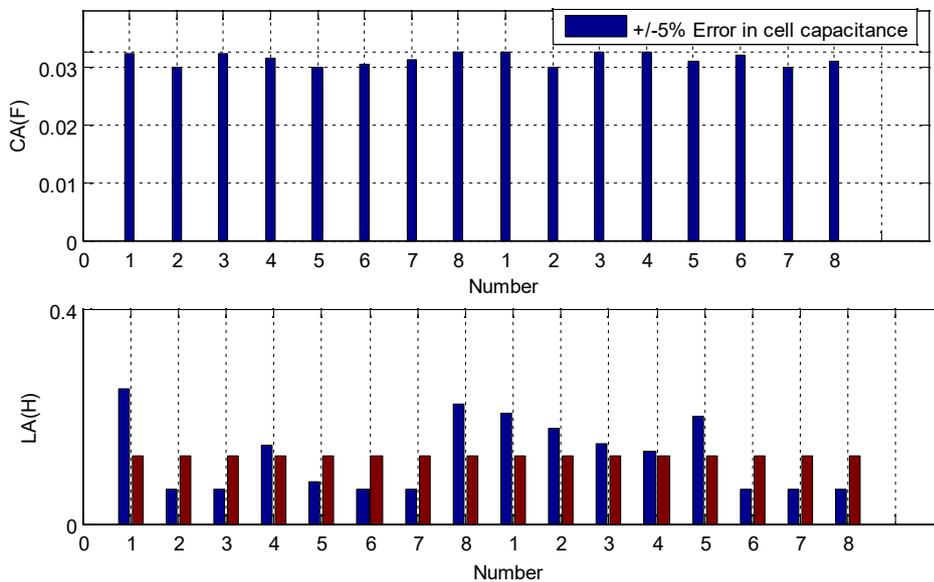


Fig.-11 Cell inductances and capacitance for configuration of two PFNs in parallel for compensating droop.

V OPTIMIZATION USING GREEN’S FUNCTION

Ordinary differential equations can be expressed in the form as,  $Lu(x) = f(x)$  where L is an ordinary linear differential operation,  $f(x)$  is a known function of independent variable x and  $u(x)$  is the desired solution. The solution to the differential equation using green’s function [5] involves finding a solution to the differential equation,  $Lg(x, \xi) = \delta(x - \xi)$ , where  $\xi$  is an arbitrary point of excitation and is Green Function. Solution  $u(x)$  is given by an integral involving the green function’s as follows.

$$u(x) = \int_{\xi}^x f(x) \cdot g(x, \xi) d\xi \text{ ---(XI)}$$

Ordinary differential equations with initial value problems can be solved using the Green function very easily. Green function can be calculated simply by forcing a delta function to the transfer function of the system.

A three-section PFN model is used to demonstrate the exact procedure of calculating compensating voltage. Using formulae in section -3, inductances and capacitances of a three-section PFN is calculated, which generates a one-second pulse and has a characteristic impedance of one ohm.

Determination of compensating voltage involves three steps. In the first step, green’s function of the network is calculated, which is nothing but a response to the transfer function of the network of the delta function for the initial value problem. The transfer function of three networks with cell inductance of 0.167 H and capacitance of 0.167 F is shown:

$$TF = \frac{130 \times s^4 + 1.9 \times 10^3 s^3 + 5 \times 10^4 s}{2.1s^6 + 130s^5 + 3.9 \times 10^2 s^4 + 1.9 \times 10^3 s^3 + 1.7 \times 10^4 s^2 + 5 \times 10^4 s + 1 \times 10^5} \text{ ---(XII)}$$

The green function is shown in Fig. 12 which is generated by forcing a delta function to the transfer function calculated above and taking inverse Laplace transform of it.

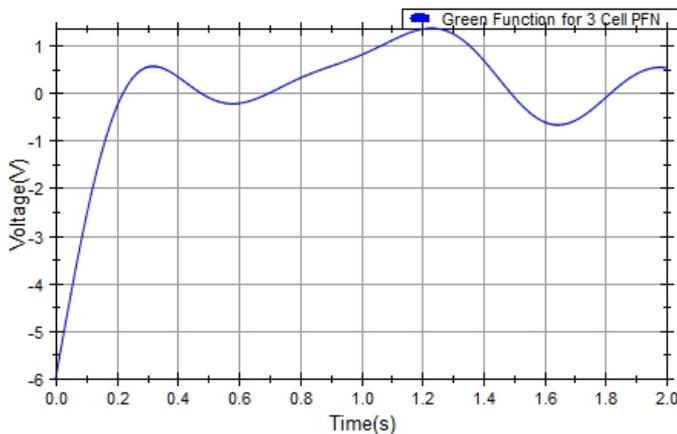


Fig. 12 Green Function for three section Network

Once the Green’s function is known, the output response of the network is calculated by forcing the input function

of the network by an integration which is the charging voltage of the network as  $V_{PFN} = \int_{\tau}^t V_c(\tau) g(t/\tau) \partial \tau \text{ ---(XIII)}$

where  $V_c$  is the charging voltage and is the at which delta function is forced to calculate Green’s Function. This response is verified by simulating the PFN using Simulink.

The second step is defining the desired pulse shape for which compensating voltage is required to be calculated. If the perfectly rectangular pulse is selected as a target, then the compensating voltage is very complex and not feasible to realize. Therefore, a pulse with flat-top with parabolic rise and fall time has been selected.

$$\left. \begin{aligned}
 Tr &= \left( 2\frac{t}{a\tau} - \frac{t^2}{a^2\tau^2} \right), 0 \leq t \leq a\tau \\
 V_p &, a\tau \leq t \leq \tau - a\tau \\
 Tf &= 1 - \left( \frac{t - \tau + a\tau^2}{a\tau} \right)^2, \tau - a\tau \leq t \leq \tau
 \end{aligned} \right\} \text{---(XIV)}$$

Where  $T_r$ , rise time,  $T_f$ , fall time,  $V_p$  is the magnitude of pulse, pulse width and  $a$  is the ratio of rise-time  $T_r$  to pulse width. Fig. 13 below shows the response of the PFN calculated using the property of green's function and the target pulse for which compensating voltage is needed.

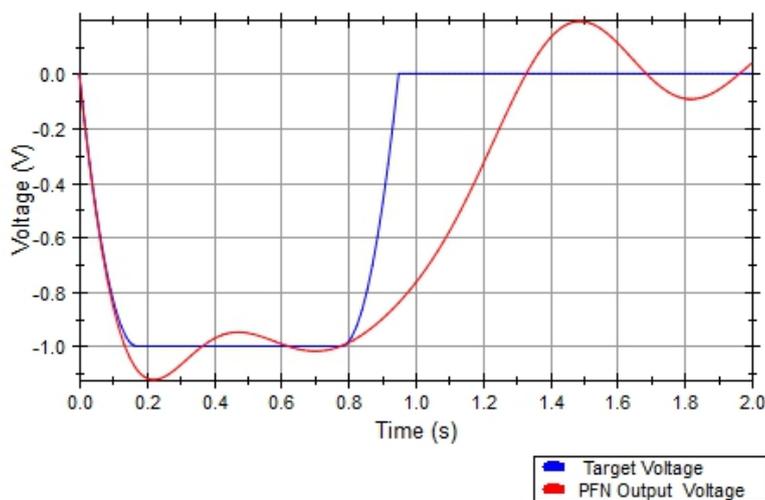


Fig. 13 The response of the PFN calculated using Green's Function and the target pulse

Finally, the input voltage is calculated by again forcing input function as desired pulse shape by integration the response is nothing but input voltage which would yield the desired output. The compensating voltage is the difference between the input voltage calculated and the charging voltage of the network input voltage. Fig.13 shows the plot of compensating voltage and the output of PFN after correction.

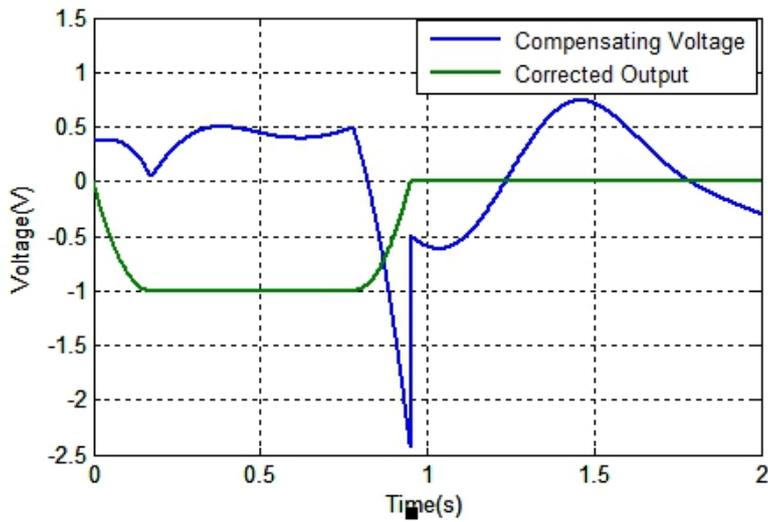


Fig.14 Compensating voltage and the output of PFN after correction.

Simulink Model used for validation of the scheme is shown in Fig. 15. Here, waveform calculated from the Matlab program was modelled using voltage controlled voltage source.

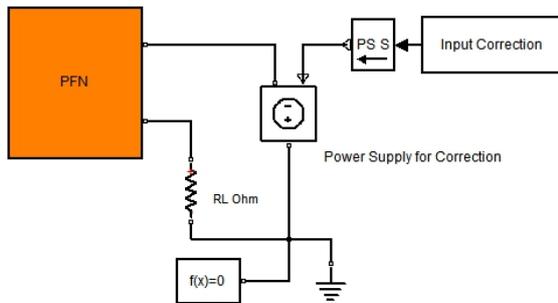


Fig. 15 Simulink Model used for validation

Since the compensating voltage is complex, it can be further simplified by considering only the part where correction is required and by increasing the number of cells in the PFN. Fig.-16 shows the output PFN of 8 cells along with the simplified compensating voltage and the corrected-out voltage.

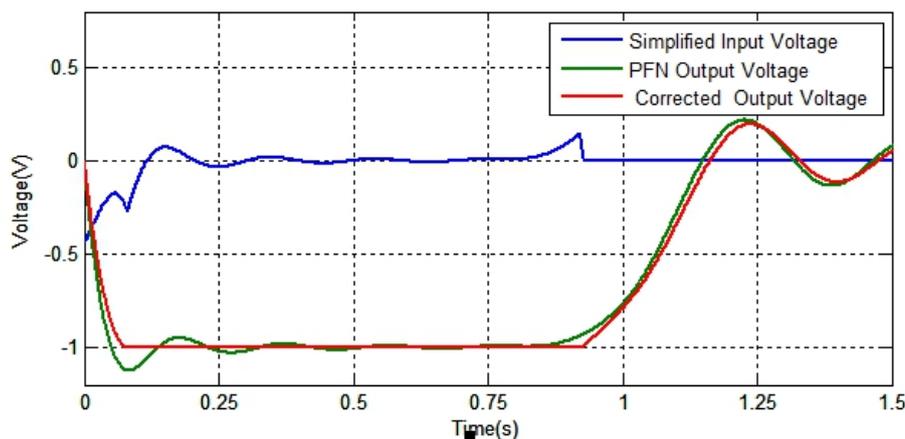


Fig. 16 PFN Output voltage of 8 cells along with the simplified compensating voltage and the corrected-out voltage.

## VI) CONCLUSION

PSO scheme has been successfully implemented to optimize the shape of PFNs. A parametric study was done with two PFN models to show that how PSO sets PFN parameters such that currents from individual currents cancel out each other ripple to generate a flat top. More than two PFNs with a larger number of cells can be made if flat top ripple requirement is critical. In another approach compensating voltage has been calculated using Green's Function. Since the magnitude of compensating voltage calculated using Green Function has a magnitude because it is just for correction. With the advent of fast SMPS technology, power supplies generating correction voltage can be easily made. It is also shown that correction can be made only on a selected part of the pulse.

## ACKNOWLEDGMENT

I thank Raja Ramanna Centre For Advanced Technology, Indore-452013 for allowing me to visit the technical area where I could study line type modulator with pulse forming networks .

## REFERENCES

- [1] Design Procedure for Compact Pulse Transformers with Rectangular Pulse Shape and Fast Rise Times D. Bortis, G. Ortiz, J.W. Kolar Power Electronic Systems Laboratory, ETH Zurich, Switzerland and J. Biela Laboratory for High Power Electronic Systems, ETH Zurich, Switzerland
- [2] Pulse Generators, Edited by G.N. Glasoe and J.V. Lebacqz
- [3] Radio Engineers' handbook by Frederick Emmons Terman, McGraw-Hill book Company, Inc
- [4] Kennedy J, Eberhart RC. Particle swarm optimization. [In: Proceedings of the International Conference on Neural Networks; Institute of Electrical and Electronics Engineers. Vol. 4. 1995. pp. 1942-1948. DOI: 10.1109/ICNN.1995.488968
- [5] A constriction factor based particle swarm optimization for economic dispatch, Shi Yao Lim, Mohammad Montakhab, and Hassan Nouri Bristol Institute of Technology, University of the West of England, Frenchay Campus, Coldharbour Lane, Bristol, BS16 1QY, United Kingdom. E-mail: [limshiyao@gmail.com](mailto:limshiyao@gmail.com)
- [6] Parameter extraction of solar cells using particle swarm optimization, Meiyang Ye,<sup>1,a</sup> Xiaodong Wang,<sup>2</sup> and Yousheng Xu<sup>1,1</sup> Department of Physics, Zhejiang Normal University, Jinhua 321004, People's Republic of China , Department of Electronic Engineering, Zhejiang Normal University, Jinhua 321004, People's Republic of China.
- [7] Improvement of the PFN control system for the klystron pulse modulator at LEBRA Nuclear Instruments and Methods in Physics Research Section A Accelerators Spectrometers Detectors and Associated Equipment 528(s 1-2):239-243 K Yokoyama I Sato