

**COMPARISON STUDY BETWEEN DIFFERENT
METHODS OF TRAVELLING WAVE INDUCTION
HEATER ANALYSIS**

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Abstract

The travelling wave induction heater (T.W.I.H) is a new and promising tool the field of heating rectangular workpieces. This heater has many advantages when compared with the existing techniques of heating such workpieces. But there is no available method or procedure to analyses this type of heaters In this paper different theories are adopted to study the and a novel equivalent circuit is introduced to predict the performance of this device A completely mathematical procedure is used in the construction of this equivalent circuit and no empirical factors were implemented.

List of Principal Symbols

A=Amperes.

a =Coefficient of the formula $B = aH^b$ of the B-H curve .

b =Coefficient of the formula $B = aH^b$ of the B-H curve .

B_m = Maximum flux density (tesla)

B_∞ =Saturated magnetic flux density (tesla).

d =Half of the workpiece thickness (m)

E_{mx} = Maximum electric field intensity in z-direction (V/m).

E_{ph} =Back e.m.f. per phase of the heater coil (volts)

f =Supply frequency (Hz)

F_n = Normal force exerted on the workpiece (newton).

g=Agap length(m)

H= Henery

H_{mx} = Maximum magnicie field intensity along the workpiece surface (A/m).

I_l =Workpiece current (A).

I_{ph} =Primary phase curent (A)

J_m =Maximum primary current sheet density (A/m)

K=Wave length factor (π/λ) .

K_w =Winding factor.

K_{tr} = Referring factor of workpiece impedance and magnetizing reactance to the primary side

m =Number of primary phases

N_{ph} =Number of series turns per phase of the heater coil.

P = Number of poles

P_w Workpiece power (watt)

R_{ph} Coil resistance per phase (Ω) .

V_{ph} =Phase voltage of the primary coil (volts).

ω =Angular frequency ($2 \cdot \pi \cdot f$).

W_c =Primary core width (m).

X_m =Magnetizing reactance (Ω).

X_{ph} =Coil leakage reactance per phase (Ω).

Z_{ph} = Primary coil phase impedance (Ω).

$Z\lambda$ =Workpiece impedance referred to the coil side (Ω).

Z_{∞} =Work-piece surface impedance (Ω)

μ =Magnetic permeability (H/m)

μ_0 =Permeability of the free space (H/m)

μ_r =Relative permeability.

λ = Pole pitch (m).

P = Electrical resistivity ($\Omega \cdot m$)

Δ = Depth of penetration (m)

$$Kb = [\sqrt{2} (1 - b)^2 / (3 + b)(1 + b)^{1/2}]^{1/2}$$

$$\delta = 2 / (1 - b)$$

$$\gamma = \sqrt{2(1 + b)} / (1 - b)$$

$$\mathfrak{S} = \sqrt{\delta^2 + \gamma^2}$$

1. INTRODUCTION

The travelling wave induction heater (T.W.I.H) is shown in figure (1). This heater overcomes many disadvantages associated with the conventional Ross coil, especially in case of heating thin workpieces, when the Ross coil becomes inefficient. Also, the (T.W.I.H) overcomes the low power factor problem of multi-layer coil. Moreover, this heater overcomes the problem of low rated power of the transverse flux induction heater, which is a single-phase load, while the (T.W.I.H) is a three-phase load [1].

Despite the numerous publications in the field of induction heating ; it is hardly to find a paper which deals with the theoretical analysis of the (T.W.I.H). In this study, different theories which can be used in the analysis of (T.W.I.H) are presented.

The performance and design parameters of the induction heating coils depend largely on the equivalent circuit technique [2]. The main disadvantage of this technique, is that, it depends on empirical factors[3].

In the present work the equivalent circuit of the (T.W.I.H) is introduced. For the construction of this circuit ; a completely mathematical technique was adopted and no empirical factors were used.

The workpiece impedance of the (T.W.I.H), which is the main parameter in the equivalent circuit, can be derived from the different theories which are presented in reference [4], to be used, in the calculation of a solid core or slab impedance, when the slab is assumed to be very thick, and the magnetic field is assumed to flow completely along the slab. These theories can be classified as linear or nonlinear, depending on the method to be used for representation of the magnetization characteristic of the workpiece material. In a linear theory the magnetic permeability is assumed to be constant. In a nonlinear theory the magnetization characteristic is represented by a suitable function while in a limiting nonlinear theory, which is a special case of the nonlinear theory, the magnetization characteristic is to be represented by a rectangular form. On the other hand these theories [4] can be classified as pulsating wave (the field quantities are function of time only) or travelling wave (the field quantities are function of time and space). The equivalent circuit is used to study the heater performance. The results, which were obtained from the different theories are compared and discussed.

2. EQUIVALENT CIRCUIT TECHNIQUE

The equivalent circuit construction is based on per phase per heater side. The methods of solid core impedance calculation are applied to calculate the workpiece surface impedance, and with the use of a referring factor (K_{tr}), the workpiece impedance, as well as the magnetizing reactance referred to the heater coil side, can be obtained to construct the equivalent circuit, by adding the coil resistance and leakage reactance. The coil resistance can be measured, or, it can be calculated, as well as the coil leakage reactance, from the linear A.C. machine theory [5], from which the factor (K_{tr}) is, also, given. These parameters of the equivalent circuit, and the heater performance calculation formulae are given in the following section.

2.1 Equivalent circuit parameters

In a double-sided (T.W.L.H) model of figure (2), due to the symmetry between the two sides, only one-side will be considered to derive its equivalent circuit. Taking the upper half, this half is divided into three regions; laminated primary core (Electrically non-conducting) , air-gap region and half of the workpiece region (conducting region). The primary coil is replaced by its equivalent current sheet density of the primary surface. Looking downward from the current sheet, there are two regions. Each region can be represented by its surface Impedance. The airgap region can be represented by a magnetizing reactance [6], while the workpiece region can be represented by the Workpiece surface impedance. These Impedances are connected in parallel and then referred to the coil side. When the coil resistance and leakage reactance are added to these parallel impedances, the equivalent circuit can be obtained, as shown in figure(3) .

From the equivalent circuit of figure (3), the coil resistance and leakage reactance are given by [5] :

$$R_{ph} = \frac{2.p_c.K_P.q.m^2.W_c.N_{ph}(1+K_1.\lambda/W_c)}{K_s.K_d.P.\lambda^2} \dots\dots\dots(1)$$

$$X_{ph} = \frac{4.\mu_0.\omega}{P} \left[\frac{(\lambda_c + \lambda_d)}{q} .w_c + \lambda_o.K_2.m/k \right] .N_{ph}^2 \dots\dots\dots(2)$$

The magnetizing reactance represents the airgap region under the current sheet, figure (2), and given by [5,6] :

$$x_m = \frac{2.f.\mu_0.\lambda^2}{\pi.g} .k_{tr} \dots\dots\dots(3)$$

The workpiece impedance referred to the coil side can be obtained as :

$$Z_l = Z_z .K_{tr} \dots\dots\dots(4)$$

Where Z_z denotes the workpiece surface impedance which is given by [4] :

$$Z_z = E_{mz} / H_{mx} \dots\dots\dots(5)$$

Where E_{mz} I denotes the maximum at electric field intensity at the workpiece surface, normal to the maximum magnetic field intensity (H_{mx}) at the workpiece surface. The factor K_{tr} is obtained from linear A.C. machine theory to be [6] :

$$K_{tr} = \frac{4.m.(N_{ph}.K_w)^2}{P.\lambda} \dots\dots\dots(6)$$

The workpiece surface impedance can be obtained from the different theories of slab impedance calculation. From each theory, the workpiece impedance, the workpiece impedance angle and the depth of penetration, can be calculated as can be seen in table (1). These values can be used to predict the heater performance, as well as the design parameters. In linear theory, the magnetic flux density, in the workpiece, is assumed to vary linearly with the magnetic field intensity, i.e. constant magnetic permeability.

In limiting nonlinear theory, the magnetic flux density is assumed to be constant at the saturated value, and the magnetic permeability depends on the magnetic field intensity only, as the B-H curve is represented by a rectangular form [7]. In nonlinear theory, the B-H curve is represented by a function, In the present work, the B-H curve of the workpiece material is represented by the formula $B = aH^b$, where the constants (a) and (b) can be found from the data points of the curve [8]. The constants (a) and (b) are determined by inspection as ($a = 0.544$) and ($b = 0.11$) to fit the curve.

2.2 Performance calculations

Once the parameters of the equivalent circuit figure (3), are known the performance of the (T.W.I.H) can be predicted in a systematic manner as :

tic manner as :

The power generated in workpiece (active power) from single-side of the heater is :

$$P = 3.I_l^2.R_l \dots\dots\dots(7)$$

The load current is:

$$I = \frac{E_{ph}}{Z_l} = (V_{ph} - I_{ph}.Z_{ph}) / Z_l \dots\dots\dots(8)$$

Where : $z_{ph} = R_{ph} + j \cdot X_{ph} \dots\dots\dots(9)$

I_{ph} can be obtained as:

$$I_{ph} = V_{ph} / [j \cdot X_m \cdot Z_l / (Z_l + j \cdot X_m) + Z_{ph}] \dots\dots\dots(10)$$

And the Primary power factor is:

$$Pf = (P_w + 3 \cdot I_{ph}^2 \cdot R_{ph}) / 3 \cdot V_{ph} \cdot I_{ph} \dots\dots\dots(11)$$

2.3 Normal force calculation

From the analytical model of figure (2), taking the upper half, which consists of same-infinite conducting region (half of the workpiece) and same-infinite nonconducting with very high permeable region (primary core). Between the two regions a current sheet which produces a traveling field and airgap. This model is coincident with that given in reference [6]. For calculation of attraction and repulsion forces, which are exerted on workpiece region. The formula to calculate these forces in this reference can be applied directly to the case of (T.W.I.H) as :

$$F_n = \frac{1}{2} P \cdot \lambda \cdot W_c \cdot \left[\frac{|B_m|^2}{\mu_0} - \mu_0 \cdot J^2 m \right] \text{ (Newton).} \dots\dots\dots(12)$$

The first term of equation (12) represents the attraction force between the primary core and the workpiece (in case of magnetic only), while the second term of this equation represents the repulsion force between the primary coil currents and the workplace currents (for both magnetic and nonmagnetic Workpieces), the net force is attraction in case of magnetic Workpieces, and repulsion for nonmagnetic Workpieces.

3. THE RESULTS OF DIFFERENT THEORIES

The present study is for thick workpieces when held by a (T.W.I.H). To study the validity of different theories of the (T.W.I.H) analysis, a comparison between these theories, for the heater performance, is developed for magnetically linear, nonmagnetic and magnetically nonlinear workpieces.

Table (2) shows the variations of phase current, Workpiece power , primary power factor and normal force with line voltage in case of travelling wave theories (Linear, Limiting nonlinear and nonlinear theories).

Table (3) shows the variations of the same parameters with line voltage, in case of pulsating wave theories (linear, limiting nonlinear and nonlinear theories). The results are obtained for magnetic workpieces(magnetically linear or nonlinear workpieces). Also **Table (4)** shows the Variations of the same parameters with line voltage in use of nonmagnetic workpiece, for travelling wave am pulsating wave linear theories.

From the comparison of these results the following points can be stated :

- i. There are very small difference between the result of (T.W.I.H) performance using travelling wave theories and Pulsating wave theories, in case of magnetically linear, nonmagnetic and magnetically nonlinear workpieces, when the pole pitch is small

compared with the heater core width ($\lambda <$ heater Core width).The maximum difference Within 5 %.

- ii. Travelling wave and pulsating wave linear theories, give large differences in the results when compared with those of limiting nonlinear and nonlinear theories, in case of magnetically nonlinear workpieces, i.e., the difference in the workpiece power is about 40 %. This means that the linear theories cannot be applied for magnetically nonlinear workpieces.
- iii. There are considerable differences in the results, between limiting nonlinear theory, when compared with that of nonlinear theory, i.e., at the same line voltage (200 volts), the difference in the phase current is about 10 % and the difference in the workpiece power is about 8 %, for both travelling wave and pulsating wave theories.

4 . EFFECT OF LARGE POLE PITCH ON THE HEATER PERFORMANCE

In the previous section, the difference between the travelling wave and pulsating wave theories was studied, and found to be very small in case of small pole pitch ($\lambda <$ heater core width). For large pole pitches ($\lambda <$ core width), the differences between the results of these theories are, also, large. The following results are obtained from the different travelling wave theories, and compared at the same line voltage, to show the effect of large pole pitches on the heater Performance.

- i) The Primary phase current increases with increasing the pole pitch, and depends on the workpiece material and the theory (method of analysis).
- ii) The workpiece power increase with increasing the pole pitch, to reach a maximum value at certain pole pitch, and then reduces with continuous increasing of the pole pitch, and the maximum value may be obtained at minimum pole pitch.
- iii) The primary power factor is slightly affected by the variation of heater pole pitch, except the case of magnetically linear workpiece, when the pole pitch is increased from 75mm to 150mm, the power factor is increased by 42%.
- iv) The normal force reduces with increasing of the pole pitch for all the methods of analysis and for all workpiece materials.
- v) There are considerable difference in the resulting, using travelling wave limiting nonlinear theory, when compared with those using travelling wave nonlinear theory. Hence, these theories should be compared with the practical measurements to show the validity of each theory in the analysis of the (T.W.I.H) .

5. COMPARISON WITH EXPERIMENTAL MEASUREMENTS

The results of the (T.W.I.H) analysis, using travelling wave limiting nonlinear theory and travelling wave nonlinear theory, are compared with those of practical measurements, which are obtained from an experimental rig, in case of steel workpiece. While the result of the travelling wave liner theory, in case of nonmagnetic (aluminium) workpiece are compared with the practical measurements.

Table (5) shows the variations of phase current, workpiece power and power factor with line voltage, in case of steel and aluminium workpieces, for similar pole arrangement (i.e. north pole in in the upper side corresponds to a north pole in the lower side).

From the comparison of these results with those given in tables (2) and (4), the following points can be stated as :

- i) The analytical results, using travelling wave nonlinear theory gives good agreement with the practical measurements than those of travelling wave limiting nonlinear theory, i.e., at $V_1 = 200$ volts, the primary phase current is lower than that of practical measurement by 6% in case of nonlinear theory, while is lower by 16% in case of limiting nonlinear theory. Also, the power factor is lower than that of practical measurement by 8% in case of nonlinear theory, while it is lower by 15 % in case of limiting nonlinear theory.
- ii) There is a close agreement between the results, using travelling wave linear theory when compared with the practical measurements, in case of nonmagnetic (aluminium) workpiece, i.e., at $V_1 = 200$ volts, the phase current is higher than that of practical measurement by 6%, the power factor is higher than that of practical measurement by 12% and the workpiece power is higher than that of practical by 16%.

6. CONCLUSION

The theories for calculation of the solid core or slab impedance, when subjected to alternating magnetic field, can be used in the analysis of the (T.W.I.H), to adopte the equivalent circuit of the heater for Performance prediction and design parameters calculations. These theories are classified as travelling wave or pulsating wave theories, and can be classified as linear, limiting nonlinear and nonlinear theories, according to the B-H curve representation.

The analysis of (T.W.I.H) is based on the equivalent circuit technique, which is a good tool in the analysis of this type of induction heaters. The workpeice impedance, which is an important parameter in the equivalent circuit, was calculated using different theories. There is a negligible difference between travelling wave and pulsating wave theories in the (T.W.I.H) analysis, at small pole pitch with relative to the heater core width, while, at large pole pitch than core width, there is a large difference between the results of the analysis, using travelling wave theories when compared with those of pulsating wave theories. This difference is continuously increased with increasing of the pole pitch, with relative to the heater core width.

Also, there is a considerable difference between the results of analysis using limiting nonlinear theory when compared with those of nonlinear theory, in case of magnetically nonlinear workpiece. These methods are compared with the practical measurements, and found that the results of nonlinear theory are more agreement with the measurements.

Moreover, the travelling wave linear theory is more suitable than the other theories in the analysis of the (T.W.I.H), in case of magnetically linear or nonmagnetic workpieces, and this method is compared with the practical measurements in case of nonmagnetic workpiece.

The difference between the analytical solutions, which are adopted in the present work using equivalent circuit technique, in case of magnetic or nonmagnetic workpieces, and the measurements, is due to the reasons that the effect of argap leakage flux, the workpiece magnetic saturation, the effect of open slotting and the saturation of the whole magnetic circuit, are not taken into account. Finally, this analysis can be used in case of thick workpieces (semi-infinite slab) and similar pole arrangement, when the field is assumed to flow completely along the workpiece (longitudinal flux heaters) .

7. REFERENCES

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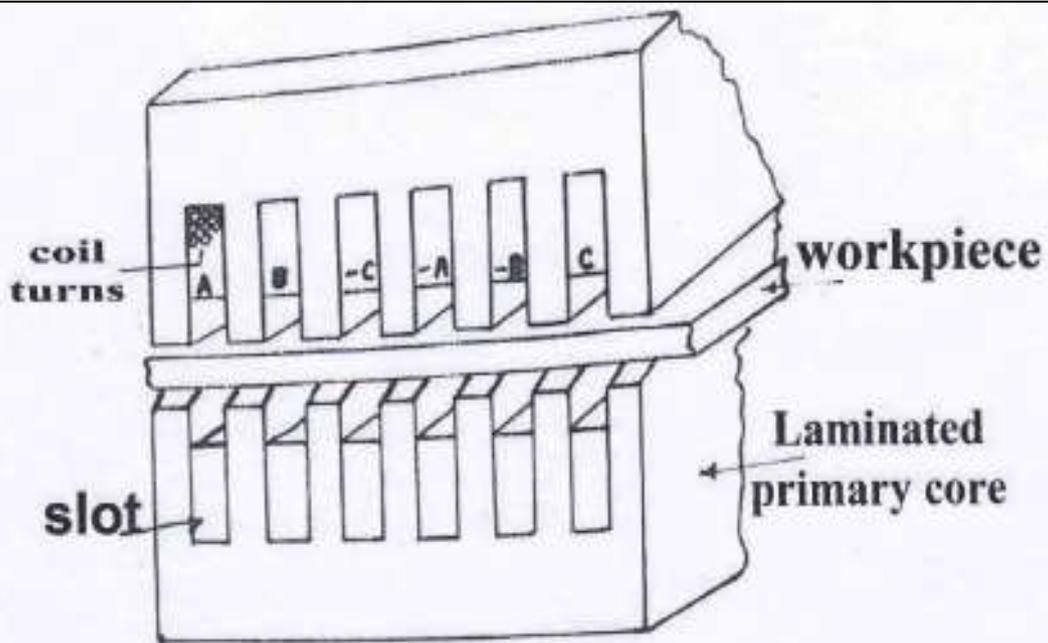
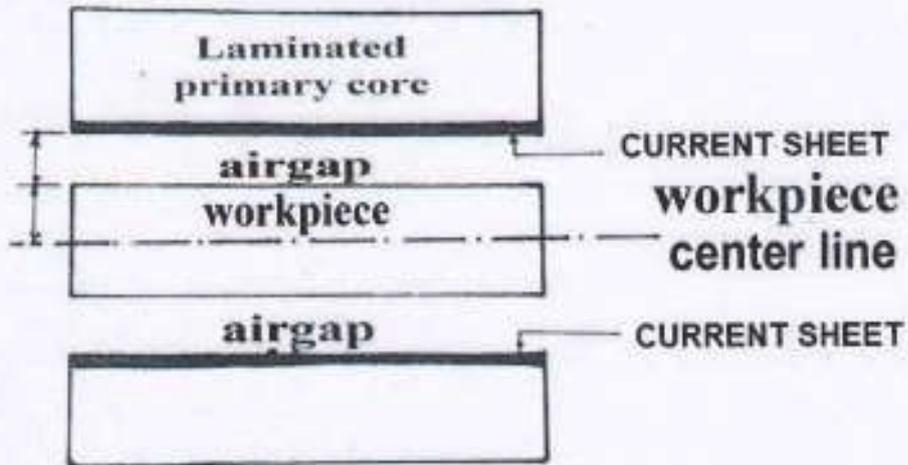


figure (1) (T.W.I.H)



Figure(2) The analytical model

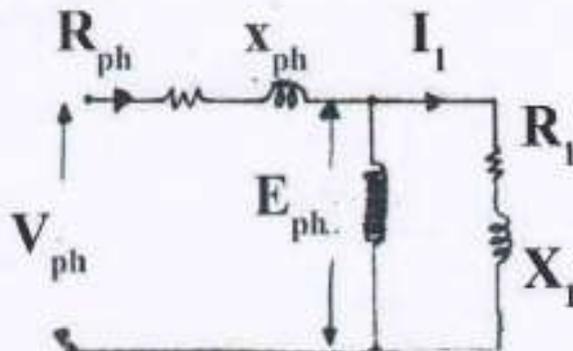


fig (3) Equivalent circuit of (T.W.I.H)

Table (1) : Workpiece parameters calculated by different theories.

Method of analysis (theories)	Skin depth(Δ)	Workpiece Surface impedance Z_z	Workpiece phase angle
Pulsating wave theory [4]	$\sqrt{2 p / w \cdot \mu}$	$\mu \cdot \varpi \cdot \Delta$	45°
Travelling wave linear theory [4]	$1 / \text{real}[(k^2 + j \cdot \omega \cdot \pi / p)^{1/2}]$	$\mu \cdot \varpi \cdot \Delta$	45°
Pulsating wave limiting nonlinear theory [7]	$\sqrt{2 H_M \cdot P / w \cdot B_\infty}$	$8 \cdot \sqrt{5 \cdot p / 3 \pi \Delta}$	26.6°
Travelling wave limiting nonlinear theory [7]	$1 / \text{real}[(k^2 + \frac{j \omega \cdot B_m}{p H_m})^{1/2}]$	$8 \cdot \sqrt{5 \cdot p / 3 \pi \Delta}$	26.6°
Pulsating wave nonlinear theory [8]	$\sqrt{2 p / w \cdot a} \cdot H_{m^{b-1}}$	$\Im \cdot k_b \cdot p / \Delta$	35.3 - 45°
Pulsating wave theory [8]	$1 / \text{real}[(k^2 + \frac{j w a H(a-b)}{p})^{1/2}]$	$\Im \cdot k_b \cdot p / \Delta$	35.3 - 45°

Table (2) Variation in phase current , workpiece power, power factor and normal force with line voltage for different travelling wave theories in case of magnetic workpieces

($\lambda = 75mm, p = 26 \times 10^{-8} \Omega \cdot m$).

Travelling wave theories for magnetic workpieces												
Line Voltage (volts)	Linear Theory*				Limiting nonlinear theory				Nonlinear theory**			
	I_{ph} (A)	P w (watt)	P.F	F_n (Newton)	I_{nh} (A)	P w (watt)	P.F	F_n (Newton)	I_{ph} (A)	P w (watt)	P.F	F_n (Newton)
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
40	4.0	35	0.32	94	3.3	25	0.33	0.3	3.5	28	0.35	0.3
80	7.3	176	0.32	315	6.8	127	0.33	405	7.5	155	0.35	340
120	10.5	260	0.32	720	10.2	343	0.33	810	12.0	400	0.36	710
160	13.8	500	0.32	1275	13.4	660	0.33	1325	15.0	760	0.36	1120
200	15.5	850	0.32	1940	16.3	1100	0.33	1870	18.0	1180	0.36	1580

* Constant magnetic relative permeability ($\mu_r = 60$), assumed.

** The B–H curve is represented by the formula $B=aH^b$,

Where $a=0.544$, and $b=0.11$ to fit the curve which has data points for steel workpiece.

Table (3) Variations in Phase current, workpiece power, power factor and normal force with line voltage for different pulsating wave theories in case of magnetic workpieces

$$\lambda = 75\text{mm}, p = 26 \times 10^{-8} \Omega.m.$$

Pulsating wave theories for magnetic workpieces												
Line Voltage (volts)	Linear Theory*				Limiting nonlinear theory				Nonlinear theory**			
	I_{ph} (A)	P w (watt)	P. F	F_n (Newton)	I_{ph} (A)	P w (watt)	P.F	F_n (Newton)	I_{ph} (A)	P w (watt)	P. F	F_n (Newton)
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
40	3.9	35	0.33	95	3.4	25	0.36	0.3	3.6	30	0.34	0.3
80	7.2	175	0.33	320	6.9	130	0.36	400	7.6	160	0.34	338
120	10.4	265	0.33	725	10.3	350	0.36	800	12.1	410	0.34	705
160	13.7	505	0.33	1282	13.5	675	0.36	1315	15.1	770	0.34	1110
200	15.4	840	0.33	1950	16.4	1125	0.36	1850	18.2	1210	0.34	1530

* Constant magnetic relative permeability ($\mu_r = 60$), assumed.

** The B–H curve is represented by the formula $B=aH^b$,

Where $a=0.544$, and $b=0.11$ to fit the curve which has data points for steel workpiece.

Table (4) Variations in Phase current, workpiece power, power factor and normal force with line voltage, for different linear theories in case of nonmagnetic (aluminum) workpieces $\lambda = 75\text{mm}$, $p = 2.8 \times 10^{-8} \Omega.m$.

Linear theories for nonmagnetic workpieces								
Line Voltage (volts)	Travelling Wave theory				Pulsating wave theory			
	I_{ph} (A)	P w (watt)	P.F	F_n (Newton)	I_{ph} (A)	P w (watt)	P.F	F_n (Newton)
0.	0.	0.	0.	0.	0.	0.	0.	0.
40	6.5	21	0.46	-15	6.4	23	0.46	-15
80	10.5	150	0.46	-95	10.4	160	0.46	-85
120	16.5	360	0.46	-262	16.3	390	0.46	-250
160	21.5	600	0.46	-535	21.0	630	0.46	-510
200	26.5	950	0.46	-930	26.2	1000	0.46	-900

Table (5) Practical Measurements:

Line Voltage V2 (volt)	Steel Workpiece			Aluminum Workpiece		
	I_{ph} (A)	P w (watt)	P.F	I_{ph} (A)	P.W	P.F
0.	0.	0.	0.	0.	0.	0.
40	4	25	0.39	5	20	0.41
80	7.8	160	0.39	10	120	0.41
120	12.5	300	0.39	15	330	0.41
160	15.6	600	0.39	20	550	0.41
200	19	1000	0.39	25	810	0.41