EVALUATION ON AGGREGATION RISK RATE FOR DEFUZZIFICATION IN FUZZY SETS

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Abstract

The main aim of the proposed method is to present that the defuzzification process using median rule is not similar to the process that follows classical centroid rule. The classical centroid rule uses either triangular or trapezoid moments. The triangular or trapezoid is taken with isosceles principle and defuzzification with such median principle is proved with reduced error tolerance.

Keywords: Fuzzification, Defuzzification, Centroid rule, Median rule, fuzzy control

1. Introduction

Fuzzy system is a logical system that is an extension of multi-valued logical system. Fuzzy logic holds the fuzzy theory sets that relate the object class with non-sharpened boundaries. Membership function is taken as a degree to identify the output values. The mapping of input to relevant output is given using fuzzy interference rule [1]. Chen [2] used median rule to defuzzify the trapezoid or triangular fuzzy inputs. Here, it is shown that the median rule of the defuzzification process is not a centroid unless it is an isosceles one. Saneifard [3] used centroid point to defuzzify the inputs using distance measurement and minimum crisp value. The methods used in [3] could be improved with isosceles principle to prove that the median and centroid with isosceles principles are same.

2. Fuzzy set theory

The fuzzy set theory introduced in [7] deals with source of vagueness problems. A modeling language is considered to approximate the situations used in fuzzy phenomena and then criteria exists. In a discourse X, a subset A of X in fuzzy is considered as a set that is defined through a membership function $f_A(x)$. This represents the mapping of each element from x in X to a [0, 1] closed interval real numbers'. The membership function value, $f_A(x)$ over a fuzzy set A is called as grade membership of x over X. The degree membership is the value of x that belongs to the fuzzy set, A. Greater the value of $f_A(x)$, the membership grade grows stronger for x in A.

The approximate reasoning is calculated from the linguistic value inside the framework of theoretical fuzzy set [8]. This handles the contingency effectively that is used in data evaluation and the also handles the vague property over linguistic expression. Further, triangular fuzzy numbers or normal trapezoid characterizes the fuzzy values of linguistic terms and quantitative data used in approximate reasoning. The fuzzy numbers are considered as the fuzzy sets with additional constraints for easier calculations. The fuzzy operations are defined using extension principle [6].

Definition 1 *Fuzzy number*: If a fuzzy set A on the universe R of real numbers satisfies the following conditions, we call it a fuzzy number.

i. A is a convex fuzzy set;

ii. there is only one x_0 that satisfies $f_A(x_0)=1$; and

iii. $f_A(x)$ is continuous in an interval.

Based on the extension principle, we can derive the arithmetic of fuzzy numbers as shown in [4-6, 9] [5].

Definition 2 *Trapezoid Fuzzy Number*: Let $\tilde{A} = (a, b, c, d)$, $a<b<c<d$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a trapezoid fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & if\ a \le x \le b \\ 1, & if\ b \le x \le c \\ \frac{d-x}{d-c}, & if\ c \le x \le d \\ 0, & otherwise \end{cases} \qquad (1)$$

Definition 3 *Triangular Fuzzy Number*: Let $\tilde{B} = (a, b, c)$, $a<b<c$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x-a}{b-a}, \ if \ a \le x \le b \\ \frac{c-x}{c-b}, \ if \ b \le x \le c \\ 0, \quad otherwise \end{cases} \quad (2)$$

Obviously, we can treat the triangular fuzzy number $\tilde{B} = (a, b, c)$ as the trapezoid (a, b, b, c).

3. Defuzzification

The fundamental reason is the indirect comparison of fuzzy numbers. The comparison over fuzzy sets has no universal consensus over several literatures. The fuzzy numbers has to be mapped initially to real values that can be compared for computing the magnitude values. The real value assignment over a fuzzy set is called as defuzzification process. The computation may take several forms; however, the standard form is the usage of centroid rule. The computation of the defuzzification process requires integrating the membership function over the fuzzy sets. This improves the effect over direct computation of centroid rule with center of gravity that describes the fuzzy quantity. The defuzzification over trapezoid or triangular fuzzy numbers using median is evaluated at the risk rate. The median is not the centroid which is shown in following rule [1]:

Proposition 1: For the trapezoid fuzzy number $\tilde{A} = (a, b, c, d)$, by the bisection of area, the median $M_{\tilde{A}} = \frac{a+b+c+d}{4}$, only if $b \le M_{\tilde{A}} \le c$.

Remark 1: For the triangular $\tilde{B} = (a, b, c)$, we can use the formula (3) treating it as the trapezoid (a, b, b, c). We have that the median of $\tilde{B}$ is $M_{\tilde{B}} = \frac{a+2b+c}{4}$.

Proposition 2: The centroid of the trapezoid fuzzy number $\tilde{A} = (a, b, c, d)$ is

$$C_{\tilde{A}} = \frac{c^2 + d^2 + cd - a^2 - b^2 - ab}{3(c + d - a - b)} \tag{3}$$

Remark 2: By the definition of the centroid of the triangular $\tilde{B} = (a, b, c)$, we obtain that the centroid of $\tilde{B}$ is $C_{\tilde{B}} = \frac{a + b + c}{3}$.

Remark 3: For the triangular $\tilde{B} = (a, b, c)$, we can use the formula (4) treating it as a trapezoid (a, b, b, c). We get the centroid of $\tilde{B}$ as $C_{\tilde{B}} = \frac{a + b + c}{3}$.

Proposition 3: The median ($M_{\tilde{A}}$) of the trapezoid equals to the centroid ($C_{\tilde{A}}$) only if the trapezoid is isosceles, i.e. *d*-*c* = *b*-*a*.

Remark 4: The median ($M_{\tilde{B}} = \frac{a + 2b + c}{4}$) of the triangular $\tilde{B} = (a, b, c)$ equals to the centroid ($C_{\tilde{B}} = \frac{a + b + c}{3}$) of the triangular $\tilde{B}$ only if the triangular is isosceles, i.e., *c*-*b* = *b*-*a*.

4. Conclusion

The proposal of the defuzzification over triangular fuzzy numbers $\tilde{B} = (a, b, c)$ using bisection of the trapezoid (*a*, *b*, *b*, *c*) is given as $C_{\tilde{B}} = \frac{a + 2b + c}{4}$ [1]. However, Proposition 3 and Remark 4, proved that defuzzifying solves the problem using median rule. The computing results could in turn eliminate the error tolerance rate unless the trapezoid or the triangular fuzzy number is isosceles.

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