

# SRSW Method for Denoising & Super-Resolution of Medical Images

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## Abstract

High resolution images are needed for many purposes. Medical imaging is one of the important applications of high resolution images. Conversion of a low resolution image to a high resolution image is the main theme. This process is called super-resolution. Non-negative quadratic programming approach is used for getting the non-negative sparse linear representation of the input patch over the low resolution patches from the database. The database contains high resolution and low resolution patch pairs. Edges of the super-resolved image can be enhanced by using the blind deconvolution technique, there by getting a sharp view of the image for treatment and diagnosis.

*Keywords: Super-Resolution, Non-negative quadratic programming, blind deconvolution, non-negative sparse linear representation*

## 1. Introduction

Image resolution is the detail an image contains. Resolution of image applies to a number of images like raster digital images, film images etc. An image with high resolution is the one which contain more image detail. Resolution depends upon the pixel size. There are many applications which are using images with high frequency. Some of the applications are astronomy, video surveillance, medical imaging etc. CT, MRI, PET and SPECT/CT are some of the technologies in digital medical imaging. The process of creating a high resolution image from a low resolution image is called super-resolution and this paper is concentrating on the creation of a high resolution image from a single noisy low resolution image.

There exist a number of limitations and it is not easy to get a desired resolution image. Environmental limitations, physical imaging system limitations, noise and blur like quality diminishing factors. Enhancing spatial resolution is an alternative solution to improving resolution. There exist some interpolation techniques for enhancing the image resolution. When the given low resolution image is corrupted by noise, the conventional

interpolation techniques become inefficient. Super-resolution is the solution for this issue.

Super resolution methods are broadly classified into two categories: multi-image super resolution and single-image super resolution. Performance of motion estimation is crucial for super-resolution, it is the limitation of the multi-image super-resolution. A new image super resolution method called “single- image super resolution” or “example-based-super resolution” came into existence. It generate the high resolution image from a single noisy low resolution image using a database of high and low resolution patch pairs.

## 2. Literature Review

In [1], Freeman et al. used a Markov network to probabilistically set up relationships between HR and paired LR patches, and between neighboring HR patches with an approximate result using belief propagation. It takes two steps to make a good quality image with the desired number of pixels and corresponding additional image details. Initially double the number of pixels in the image, by means of a conventional image interpolation method such as cubic-spline

or bilinear interpolation. Then, calculate missing image details in the interpolated image to make the super-resolution output.

In [2], Chang et al. proposed to resolve the HR patch based on a linear combination of HR patches. This method uses the LLE (Locally Linear Embedding) method. For that, after finding the linear combination of the nearest LR neighbors such that it is nearby to a given input LR patch, the output HR patch is anticipated by replacing LR patches with the associated HR patches in the linear combination of nearest neighbours.

In [3], Kim et al. used the connection between HR and LR patch pairs based on a regression function. During the super-resolution phase, corresponding patch of the given low-resolution image is checked with the stored low resolution patches, then the high-resolution patch equivalent to the nearest low-resolution patch is selected as the output.

In [4], J Yang et al. projected a single-image super-resolution, based on sparse signal representation. Training two dictionaries for the low- and high-resolution image patches, and can enforce the similarity of sparse representations connecting the low resolution and high resolution image patch pair with respect to their own dictionaries.

### 3. SRSW Method

Construct normalized examples of high/low-resolution image patch pairs. In the HR patch step, it is understood that each HR patch can be calculated as a sparse non-negative linear combination of the standard HR patches in the database. To create each HR patch from a given LR patch, the plan lies in finding the most reliable example HR patches from the database. The LR image  $Y$  will be shown as a set of  $N$  overlapping image patches, that is,  $Y = \{y_i^l, i = 1, 2, \dots, N\}$ . A set of  $N$ , HR patches represent the high resolution image  $X$ ,  $\{x_i^h, i = 1, 2, \dots, N\}$ . Here  $y_i^l$  is a patch of size  $\sqrt{m} \times \sqrt{m}$  and  $x_i^h$  is put to be  $\sqrt{n} \times \sqrt{n}$ .

The association between the LR patch and HR patch are,  $y_i^l = D_s H x_i^h + \eta_i$

Here  $D_s$  is the decimation operator with a factor  $s$ ,  $H$  is the blur operator, and  $\eta$  is the additive noise component.

This scheme contains two steps. They are database construction and super resolution construction. Standard images used to construct the database is as shown in figure 1.

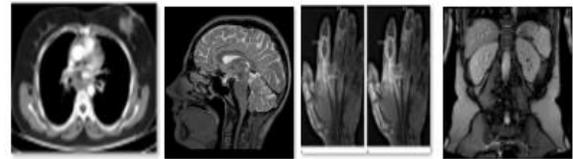


Fig. 1 Standard images used in database

With the example images, a set  $\{p_k^h, k \in I\}$  of vectorized image patches of size  $\sqrt{n} \times \sqrt{n}$  is first selected. After that, in support of each  $p_k^h$ , an equivalent vectorized patch  $p_k^l \in R^m$  is come across by,  $p_k^l = D_s H p_k^h$ . Think about  $p_k^h$  as a HR patch and  $p_k^l$  as the equivalent LR version.

The database of patch pairs is shown as below,

$$(P_l, P_h) = \{(u_k^l, u_k^h) = \left( \frac{p_k^l}{\|p_k^l\|}, \frac{p_k^h}{\|p_k^h\|} \right), k \in I\} \quad (1)$$

Determine the HR patch  $x_i^h$ , represented by  $x_i^h$ , from  $y_i^l$  by using the database  $(P_l, P_h)$

With sparse non-negative linear combination of the HR patches in  $p_h$ , find out the HR patch,

$$x_i^h = \sum_{k \in I} \alpha_{ik} u_k^h \quad (2)$$

The sparse representation vector can be in the form,  $\alpha^i = [\alpha_{i1}, \alpha_{i2}, \dots]^T \geq 0$

Find out the sparse representation vector  $\alpha^i$  of  $y_i^l$  in regard of LR patches  $P_l$ . Then multiply this representation by the database  $P_h$ ,

$$x_i^h = P_h \alpha^i$$

Sparse representation vector can be,

$$\alpha^i = \arg \min_{\alpha \geq 0} \|\alpha\|_0 + \sum_{k \in I} w_{ik} a_{ik} \quad (3)$$

Here,  $\alpha = [\alpha_{i1}, \dots, \alpha_{ik}, \dots]^T$ ,  $\epsilon$  is a given positive number,  $\sigma_i$  is the standard deviation of the noise in the  $i$ -th patch. By using their LR versions  $y_i^l$  and  $u_k^l$  the dissimilarity between  $x_i^h$  and  $u_k^h$  is calculated. The positive penalty coefficients  $w_{ik}$  linked with the dissimilarity. Penalty coefficient is specified by,

$$w_{ik} = \phi_i(d(y_i^l, u_k^l)) \quad (4)$$

Here  $d: R^m \times R^m \rightarrow R^+$  is a condition measuring the dissimilarity between  $y_i^l$  and  $u_k^l$

In case of congruence of image patches the constant  $\mu_{ik} > 0$ .

$$y_i^l = \mu_{ik} u_k^l + \eta_i \quad (5)$$

The mean of  $\eta_i$ ,  $E(\eta_i) \approx 0$ . So,

$$E(y_i^l) = \mu_{ik} E(u_k^l) + E(\eta_i) \Rightarrow \mu_{ik} = \frac{E(y_i^l)}{E(u_k^l)} \quad (6)$$

Statistical property of noise is calculated with the parameter  $a_{ik}$ .

$$a_{ik} = |E(y_i^l - \mu_{ik} u_k^l)| + |Var(y_i^l - \mu_{ik} u_k^l) - \sigma_i^2| \approx 0 \quad (7)$$

The dissimilarity measure 'd' is given by,

$$d(y_i^l, u_k^l) = \|y_i^l - \mu_{ik} u_k^l\|_2^2 + a_{ik} \quad (8)$$

In the ideal case,

$$d(y_i^l, u_k^l) \approx \|\eta_i\|^2 \approx \gamma(m\sigma_i^2) \quad (9)$$

Here 'm' is the number of elements in vector  $y_i^l$  ( $y_i^l \in R^m$ ), and 'γ' is a positive constant. The value of  $\alpha^i$  is get by using the following algorithm,

### Multiplicative Updates Algorithm for NQP

Input:  $\alpha = \alpha_0 > 0$ ,

Updating:  $t=0$

While  $t < T$  &  $\|y_i^l - U_i \alpha_t\|_2^2 > m\sigma_i^2$

$$\alpha_{t+1} = \alpha_t \cdot * (U_i^T y_i^l) ./ (U_i^T U_i \alpha_t + w_i)$$

T=t+1

End

Output:  $\alpha^i = \alpha_t$

The HR patch  $x_i^h$  can be measured as,

$$x_i^h = \sum_{k \in I} \alpha_{ik} u_k^h \quad (10)$$

The denoised version of the image is ,

$$y_i^l = U_i \alpha^i = \sum_{k \in I_i} \alpha_{ik} u_k^l \quad (11)$$

The whole HR image is get by,

$$\min_X \left\| X - X^{\Lambda}_{coarse} \right\|_2^2 \tag{12}$$

Make use of the iterative back-projection algorithm to resolve this problem:

$$X_{t+1} = X_t + ((Y^{denoise} - D_s H X_t) \uparrow s) * P \tag{13}$$

Here  $X_t$  is the estimate of the HR image at the t-th iteration,  $\uparrow s$  denotes up-scaling by factor s,  $P$  is a symmetric Gaussian filter. Averaging overlapping regions produce  $X^{\Lambda}_{coarse}$ . Similarly  $Y^{denoise}$ .

The complete Super-Resolution algorithm is ,

### Super-Resolution Algorithm

INPUT:

- Low resolution image Y and the patch size of low resolution image  $\sqrt{m} \times \sqrt{m}$
- Magnification factor s
- Database  $(P_l, P_h) = \{(u_k^l, u_k^h), k \in I\}$  ,  
 $P_l \in R^{m \times M}$  ,  $P_h \in R^{n \times M}$
- Regularization parameter  $\lambda$ , number of iterations T

OUTPUT: HR image  $X^{\Lambda}_{final}$

BEGIN

1. Partition Y into arranged set of N overlapping pathes of size  $\sqrt{m} \times \sqrt{m}$  ,  
 $\{y_i^l\}_{i=1}^N$
2. For each patch  $y_i^l$  ,

a) Using (6), (7) and (8) compute the dissimilarity criteria,  $d(y_i^l, u_k^l)$

b) Determine the subset  $I_i$

c) Using If  $\sigma > 0$ , compute the penalty coefficient  $w_i$

d) Solve the problem (10) using multiplicative updates algorithm.

e) Generate the high resolution patch  $x_i^{\Lambda_h} = \sum_{k \in I} \alpha_{ik} u_k^h$  and the denoised version of patch

$$y_i^{\Lambda_l} = U_i \alpha^i .$$

3. End

4. Fusion: Produce the initial HR image  $X^{\Lambda}_{coarse}$  and the denoised image  $Y^{denoise}$ .

5. IBP enhancement: Construct the final HR image  $X^{\Lambda}_{final}$  using the IBP procedure on the coarse HR  $X^{\Lambda}_{coarse}$

END

The super-resolved image can be enhanced by using the blind deconvolution algorithm of edge enhancement. Blind deconvolution is the generation of a sharp version of a blurred image when the blur kernel is unknown.

### 4. Implementation

Denosing is one of the important part of the image processing. In some cases the image can be corrupted by noise. So first denoise the image and then do the super resolution.

#### 4.1 Low Resolution Image Creation

Low resolution image can be created by a combination of smoothing ( 7x7 gaussian filter)

and downsampling (with a decimation factor of 's') operations and then do the noise addition. Default size of the low resolution patch is 5 x 5 and 9x9 for HR patch. Figure 2 shows the LR patches.

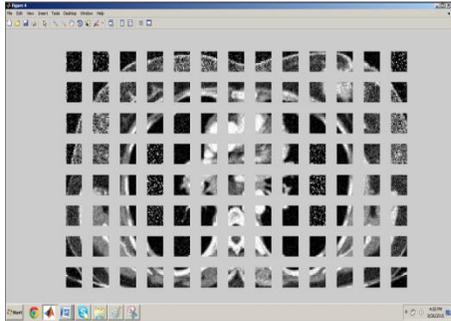


Figure 2: LR image patches

The created low resolution image is as shown in figure 3.

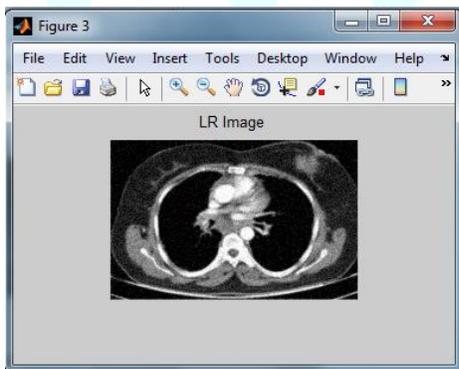


Figure 3: LR image (chest)

#### 4.2 Database Creation

The database contains both high and low resolution patch pairs. The equation for creating the low resolution patch is,

$$p_k^l = D_s H p_k^h \quad (14)$$

#### 4.3 HR Image Creation

In this step, the super-resolution and denoising on image patch is performed with sparse weight optimization model. It has to use the super resolution algorithm to get the entire high resolution image.

#### 4.4 Edge Enhancement

Blind deconvolution is used to enhance the image resolution. Input blurry images form a matrix with minimum eigen vector it is equal to blur kernel. With the blurred image, it create a sharp image and a blur kernel.

### 5. Results

CT and MRI images are used for conducting the experiments. To demonstrate the efficiency of the SRSW method results of the MRI image of brain with noise level  $\sigma = 10$  is shown in the figure 5.1.

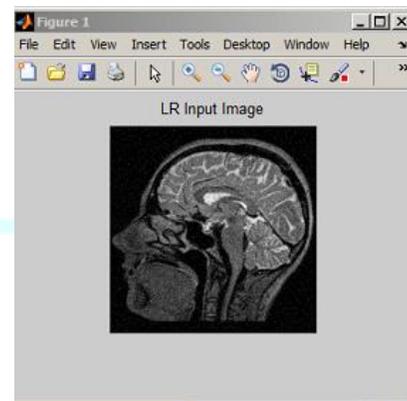


Figure 4: LR image (brain)

To deal with noisy data, this method uses the incorporated framework of denoising and super-resolution of medical images. Denoising is desired to conserve the edge details. Figure 5 shows the denoised brain image.

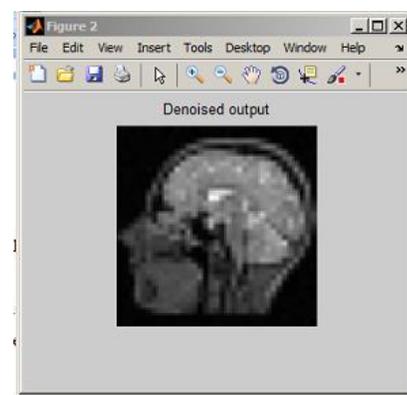


Figure 5: Denoised image

To create the denoised image, the noisy patches are replaced by the denoised patches. By using this denoised output, its super resolution output is generated, it is shown in figure 6. From the super resolution output, it is possible to generate the super resolved image with more sharp edges by using the blind deconvolution algorithm (figure 7).

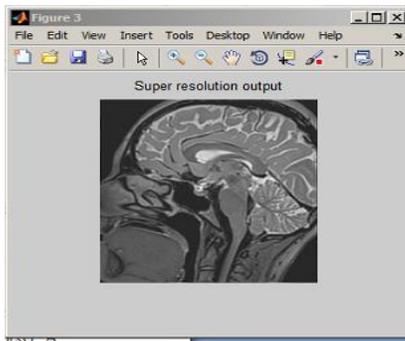


Figure 6: Super-resolution output

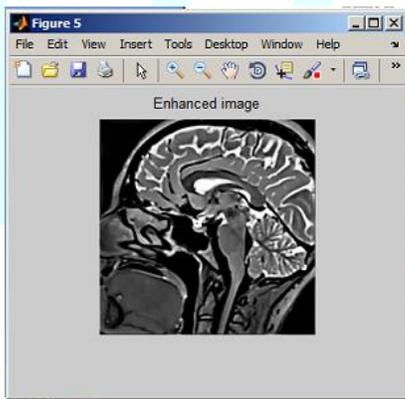


Figure 7: Enhanced image

## 6. Conclusion

SRSW method for denoising and super resolution of medical images is very helpful for noisy and low resolution images. Example based standard images are used for database creation. This method is based on the sparse positive linear representation of HR patches. The super resolved image is enhanced by using the blind deconvolution algorithm, there by getting a more sharp image for treatment and diagnosis.

## References

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