Passive Vibrations Control of Ultrasonic Machining Subjected to Tuned and External Forces

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Abstract

Linear passive control is used to study and description the vibration of the ultrasonic machining (USM) non-linear dynamical system is consists of tuned and external force .The system is represented by a two-degree-of-freedom (2DOF).The method of multiple scales perturbation technique (MSPT) is applied to obtain approximate solution. The vibration is studded numerically on the system with and without control .The stability at the worst resonance cases ($\Omega_1 = \omega_1, \Omega_1 = \omega_2, \omega_1 \cong \omega_2, \omega_1 = 2\omega_2,\ \Omega_2 = \omega_1 \pm \Omega_3, \Omega_2 = \omega_2 \pm \Omega_3$) is obtained using both phase plane methods and frequency response of resonance case. Effects of different parameters on the system behavior are studied numerically by using MATLAB program. Finally, a comparison of previously published work is done.

Keywords: Stability, Frequency response, Multiple times scale, Vibration control, tuned force, excitation, Passive control.

1 Introduction

Passive control was applied to reduce the vibration of the (USM) applying nonlinear absorber (tool) (which represents many applications in machine tools, ultrasonic cutting process). The advantages of using absorber (passive control), which leads to time saving and higher machining efficiency. And the use of active feedback control strategy is a common way to stabilize and control dangerous vibrations in vibrating systems and structures, such as bridges, highways, buildings, space and aircrafts

Hamed et al [1] introduced a mathematical and numerical study on controlling the nonlinear response of vibrational vertical conveyor under mixed excitation. By studying the vibrating motion of vertical vibration conveyor, we wrote the equations of motion as a coupling of non-linear differential equations. Multiple scale perturbation method is applied to study the approximate mathematical solutions up to the second order approximation and we studied the stability of the steady state solution mathematically at the worst different resonance cases using frequency response equations. Alısverisci [2] study the transitional behavior across resonance, during the starting of a single degree of freedom vibratory system excited by crank and-rod. A loaded vibratory conveyor is safer to start than an empty one. Shaking conveyers with cubic nonlinear spring and ideal vibration exciter have been analyzed analytically for primary resonance by the Method of Multiple Scales, and numerically. The approximate analytical results obtained in this study have been compared with the numerical results, and have been found to be well matched.

Yuejing Zhao et al [3] conducted the configuration and force analysis of vertical vibratory conveyor. The model of system with considering the friction between the materials and the spiral conveying trough is developed. The numerical simulations are done and the dynamical responses curves are given. Suitable configuration parameters of vertical vibratory conveyor and parameters of materials can make it work normally. Hüseyin Bayıroğlu [4] analyzed the nonlinear analysis of unbalanced mass of vertical conveyor with non-ideal DC motor. The results of numerical simulation are plotted and Lyapunov exponents are calculated. El-Sayed and Bauomy [5] used the two positive position feedback controllers (PPF) are used to reduce the vertical vibration in the vertical conveyors. An investigation is presented of the response of a four degree-of-freedom system (4-DOF) with cubic nonlinearities and external excitations at primary resonance. Hamed et al. [6] investigated the nonlinear vibrations and stability of the MEMS gyroscope subjected to different types of parametric excitations. The averaging method is applied to obtain the frequency response equations for the case of sub-harmonic resonance in the presence of 1:1 internal resonances. The stability of the system is investigated with frequency response curves and phase-plane method.

Sayed et al [7] find analytical and numerical study to investigate the vibration and stability of the Van der Pol equation subjected to external and parametric excitation forces via feedback control. The stability of the system is investigated applying Lyapunov first method. The stability of the system is investigated applying Lyapunov first method. Gulyayev and Tolbatov [8] and Dai and Dong [9].They wanted to prove that Active control is now commercially available for reducing vibrations offering better comfort with less weight than traditional passive technologies. The Van der Pol equation is of great interest because it can serves as a basic model for self-excited oscillations in many disciplines. Warminski et al. [10] discussed active suppression of nonlinear composite beam vibrations by selected control algorithms. Wang et al. [11] presented theoretical and experimental study of active vibration control of a flexible cantilever beam using piezoelectric actuators.

El-Ganaini et al**.** [12] applied positive position feedback active controller to reduce the vibration of a nonlinear system. They found that the analytical and numerical solutions are in good agreement**.** Eissa et al. **[**13] applied a proportional-derivative controller to the nonlinear magnetic levitation system subjected to external and parametric excitations. They studied the effects of proportional and derivative gains to give the best performance for the system. El-Ganaini [14]. Investigates the vibration control of a harmonically excited Duffing oscillator via a simple pendulum. The amplitude-phase modulating equations governing the system dynamics are extracted utilizing perturbation methods. Bifurcation analyses are conducted and the Lyapunov direct method is applied to study the system stability. The uncontrolled system exhibits a variety of nonlinear phenomena such as jump phenomenon, saddle-node, and trans critical bifurcations. Warminski et al. [15] studied vibration analysis of an autoparametric pendulum-like mechanism subjected to harmonic excitation. They proposed a suspension composed of a semi-active MR damper and a nonlinear spring. Kecik [16] studied the nonlinear oscillations of autoparametric system consists of a nonlinear oscillator attached to pendulum system.

Tusset et al. [17] studied the chaotic behaviors control of parametrically excited pendulum using two different control strategies. One of this applied control method is via the active nonlinear saturation controller, and the other via introducing a passive rotational MR damper. Hamed et al [18] investigated the effects of an active vibration control on a nonlinear two-degree-of-freedom system described by a nonlinear differential equations subjected to mixed excitation forces. The method of multiple scale perturbation technique is applied to determine the approximate solutions of the coupled nonlinear differential equations up to the second order approximation. The frequency response equations and phase plane technique at the worst resonance cases are used to study the stability of the vibrating system. Yabuno et al. [19] proposed a non-linear active cancellation method to stabilize the principal parametric resonance in a magnetically levitated body subjected to an unsymmetrical restoring force. Jun et al. [20], introduced the non-linear saturation-based control strategy for the suppression of the self-excited vibration of a van der Pol oscillator. It is demonstrated that the saturation-based control method is effective in reducing the vibration response of the self-excited plant when the absorber's frequency is exactly tuned to one-half the natural frequency of the plant. Jun et al [21], studied An active non-linear vibration absorber for suppressing the high amplitude vibration of the non-linear plant when subjected to primary external. The absorber is based on the saturation phenomenon associated with the dynamical systems with quadratic non-linearities and 2:1 internal resonance.

Hamed et al. [22] investigated the nonlinear vibrations, energy transfer and stability of the MEMS gyroscope system under multi-parametric excitations. Also, they obtained the frequency response equations using the averaging method. Abdelhafez and Nassar [23]. They studied Loop delays and They took into consideration at ion when positive position feedback controller is used to control the vibrations of forced and self-excited nonlinear beam. External excitation is a harmonic excitation caused by support motion of the cantilever beam. Self-excitation is caused by fluid flow and modeled by a nonlinear damping with a negative linear part (Rayleigh's function). Gao and Chen [24] have been studied extensively the vibration control of many systems with the time delay by using different controllers. An active linear absorber based on positive position feedback control strategy has been developed and applied to suppress the high-amplitude response of a flexible beam subjected to a primary external excitation**.** Shin et al [25]. They achieved control of the active vibrations in the clamps using the control unit in the positive feedback of the position with the torque sensor pair operator. Yingli et al [26] studied Dynamic effects of delayed feedback control on nonlinear vibration floating raft systems.

Gao, and Chen [27] studied Nonlinear analysis, design and vibration isolation for a bilinear system with time-delayed cubic velocity feedback. Abdelhafez and Nassar [28] presented a quantitative analysis on the nonlinear behavior of a forced and self-excited beam coupled with a positive position feedback controller PPF. Such that the external excitation is a harmonic motion on the support of the cantilever beam. Self-excitation is caused by fluid flow and modeled by a nonlinear damping with a negative linear part (Rayleigh's function). Self-excitation can build up oscillations even in the absence of external forces. Also self-excitation can interact with the external excitation and lead system to vibrate with a quasi-periodic motion and to be unstable. This problem is treated here by using PPF controller. Jun et al. [29] applied a nonlinear saturation controller NSC with van der Pol oscillator and additionally investigated the influence of feedback gains by using perturbation and direct numerical integration solutions. Ouakad et al [30], the behavior of a micro beam is improved by using a nonlinear feedback controller. Also authors presented a novel control design that regulates the pass band of the considered micro beam.

Jian et al [31] improved a nonlinear saturation controller and utilized it to reduce high-amplitude vibrations of a flexible, geometrically nonlinear beam-like structure. In this article we study effect of passive control on the system and ability to control on vibration .The method of multiple time scales is perturbation technique is applied to obtain the periodic response equation near the selected resonance case. Yan et al. [32] they have verified the possibility of a vehicle suspension system under time delayed the optimal control. Showed how to effect of time delay on control stability of the action on the system. Have used the mathematical simulation to verify the rightness of the stable interval obtained by differential equation theory for linear systems with constant coefficients and time delay.

Di Ferdinando and Pepe. [33] they discussed the problems of the stabilization in the sample-and-hold sense by simulation of continuous-time dynamic produce feedback controllers, so They studied the time delay of the nonlinear systems and appropriate conditions are provided such that the simulation of continuous-time dynamic output feedback controller. In this paper the (MSPT) is applied to obtain approximate solution. Studying the vibration numerically on the system with and without control, and we discuss the effect of resonance cases resulting from different force.

2. Mathematical modeling

The non-linear dynamical system is consists of parametric and external force .The system is represented by a two-degree-of-freedom (2dof) differential equations represented by the main system and absorber. From the principles of the mechanics the derived equation of motion can be written the forms Eqs (1) and (2) as shown in [34].

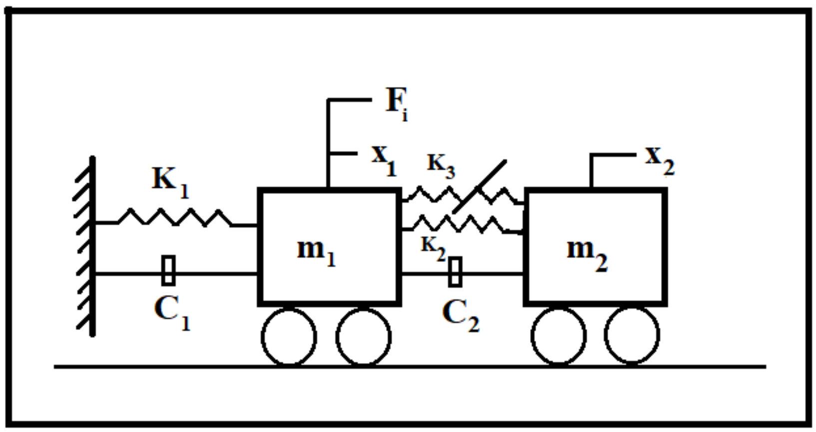


Figure1 Schematic graph of ultrasonic machine

where m_1 and m_2 are the mass of the main system and the absorber. c_1 and c_2 are damping coefficients of the main system and the absorber. k_1, k_2 and k_3 are stiffness of the main system and the absorber. F is excitation amplitude of tuned and external force.

$$\ddot{x}_1+\omega_1^2 x_1+\varepsilon\alpha_1(\dot{x}_1-\dot{x}_2)+\varepsilon\alpha_3\dot{x}_1+\varepsilon\alpha_4(x_1-x_2)+\varepsilon\alpha_5(x_1-x_2)^2=\varepsilon f_1\cos\Omega_1 t$$
$$+\varepsilon f_2\cos\Omega_2 t\sin\Omega_3 t \quad (1)$$
$$\ddot{x}_2+\varepsilon\alpha_2(\dot{x}_2-\dot{x}_1)+\omega_2^2(x_2-x_1)-\varepsilon\alpha_6(x_1-x_2)^2=0 \quad (2)$$

where x_1, x_2 are the displacements of the linear oscillators and nonlinear energy sink (NES), $\ddot{x}_1$ $\ddot{x}_2$ derivatives of x_1, x_2, (α_1, α_3) are the damping coefficient, εf_i *(i=1, 2)* are the amplitudes of excitation of each linear oscillator, ε is a small parameter and $\delta, \alpha_7, \alpha_8$ are defined in appendix , ω_1, ω_2 are natural frequencies, $\Omega_1, \Omega_2, \Omega_3$ are forcing frequency . Let $u = x_1$ and $v = x_2 - x_1$ then, equations (1) and (2) can be written as:

$$\ddot{u}+\omega_1^2 u-\varepsilon\alpha_1\dot{v}+\varepsilon\alpha_3\dot{u}-\varepsilon\alpha_4 v+\varepsilon\alpha_5 v^2=\varepsilon f_1\cos\Omega_1 t+\varepsilon f_2\cos\Omega_2 t\sin\Omega_3 t \quad (3)$$

$$\ddot{v}+\omega_2^2 v+\varepsilon\left(-\delta\omega_1^2 u-\alpha_3\dot{u}+\alpha_4 v+\alpha_7\dot{v}-\alpha_8 v^2\right)=-\varepsilon f_1\cos\Omega_1 t-\varepsilon f_2\cos\Omega_2 t\sin\Omega_3 t \tag{4}$$

2.1 Perturbation analysis

Eqs (3) and (4) can be solved analytically using multiple time scale perturbation technique as:

$$u(T_0,T_1)=u_0(T_0,T_1)+\varepsilon u_1(T_0,T_1)+\varepsilon^2 u_2(T_0,T_1)+\dots \tag{5}$$

$$v(T_0,T_1)=v_0(T_0,T_1)+\varepsilon v_1(T_0,T_1)+\varepsilon^2 v_2(T_0,T_1)+\dots \tag{6}$$

where T_0=t is fast time scale, which is associated with changes occurring at the frequencies, $\Omega_1,\Omega_2,\Omega_3$.and T_1= εt is the slow time scale, which is associated with modulations in the amplitudes and phases resulting from the non-linearity's and parametric resonance. In term of T_0 and T_1 the time derivatives became

$$\frac{d}{dt}=D_0+\varepsilon D_1+\varepsilon^2 D_2+\dots \tag{7}$$

$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2(D_1^2+2D_0 D_2)+\dots \tag{8}$$

Where D_n differential operators; $D_n=\frac{\partial}{\partial T_n}$ (n=0, 1). Substituting Eqs (5) and (6) into Eqs (3) and (4) and equation the coefficients of same power of ε in both sides, we obtain:

Order (ε^0): $(D_0^2+\omega_1^2)u_0=0$ (9)

$$(D_0^2+\omega_2^2)v_0=0 \tag{10}$$

Order (ε^1):

$$(D_0^2+\omega_1^2)u_1=f_1\cos\Omega_1 t+f_2\cos\Omega_2 t\sin\Omega_3 t-2D_0D_1u_0-\alpha_3 D_0u_0+\alpha_4 v_0$$
$$+\alpha_1 D_0 v_0-\alpha_5 v_0^2 \tag{11}$$

$$(D_0^2+\omega_2^2)v_1=-2D_0D_1v_0+\delta\omega_1^2 u_0+\alpha_3 D_0 u_0-\alpha_4 v_0-\alpha_7 D_0 v_0+\alpha_8 v_0$$
$$-f_1\cos\Omega_1 t-f_2\cos\Omega_2 t\sin\Omega_3 t \tag{12}$$

The general solution of Eqs (9) and (10) can be expressed in the form:

$$u_0(T_0,T_1)=A_0 e^{i\omega_1 T_0}+cc \tag{13}$$

$$v_0(T_0,T_1)=B_0 e^{i\omega_2 T_0}+cc \tag{14}$$

where A_0 and B_0 are unknown function in T_1, which can be determined by imposing the solvability condition at the next approximation order by eliminating the secular and small-divisor terms.

Substituting Eqs (13) and (14) into Eqs (11)and (12) we get:

$$(D_0^2+\omega_1^2)u_1=\frac{f_1}{2}(e^{i\Omega_1T_0}+e^{-i\Omega_1T})-\frac{if_2}{4}(e^{i(\Omega_2+\Omega_3)T_0}-e^{i(\Omega_2-\Omega_3)T_0})-2i\omega_1D_1A_0e^{i\omega_1T_0}$$
$$-i\alpha_3\omega_1A_0e^{i\omega_1T_0}+\alpha_4B_0e^{i\omega_2T_0}+i\omega_2\alpha_1B_0e^{i\omega_2T_0}-\alpha_5B_0^2e^{2i\omega_2T_0} \quad (15)$$

$$(D_0^2+\omega_2^2)v_1=-\frac{f_1}{2}(e^{i\Omega_1T_0}+e^{-i\Omega_1T})+\frac{if_2}{4}(e^{i(\Omega_2+\Omega_3)T_0}-e^{i(\Omega_2-\Omega_3)T_0})-2i\omega_2D_1B_0e^{i\omega_2T_0}$$
$$+i\delta\omega_1^2A_0e^{i\omega_1T_0}-\alpha_4B_0e^{i\omega_2T_0}+i\omega_1\alpha_3A_0e^{i\omega_1T_0}+\alpha_8B_0e^{i\omega_2T_0}-i\omega_2\alpha_7B_0e^{i\omega_2T_0} \quad (16)$$

Eliminating the secular terms, the general solution of Eqs (15) and (16) are given by:

$$u_1(T_0,T_1)=A_1e^{i\omega_1T_0}+\Gamma_1e^{i\omega_2T_0}+\Gamma_2e^{2i\omega_2T_0}+\Gamma_3e^{i\Omega_1T_0}+\Gamma_4e^{i(\Omega_2+\Omega_3)T_0}+\Gamma_5e^{i(\Omega_2-\Omega_3)T_0} \quad (17)$$

$$v_1(T_0,T_1)=B_1e^{i\omega_2T_0}+\Gamma_6e^{i\omega_1T_0}+\Gamma_7e^{i\Omega_1T_0}+\Gamma_8e^{i(\Omega_2+\Omega_3)T_0}+\Gamma_9e^{i(\Omega_2-\Omega_3)T_0} \quad (18)$$

where (Γ_i , i=1…8) and A_1, B_1 are complex function in T_1, and cc is complex conjugate of the preceding terms.

From the analytic solution the possible resonance cases are :

(1) Primary resonance $(\Omega_1=\omega_1)$ $(\Omega_1=\omega_2)$

(2) Internal resonance $(\omega_1\cong\omega_2)$

(3) Combined resonance $(\Omega_2=\omega_1\pm\Omega_3)$ $(\Omega_2=\omega_2\pm\Omega_3)$

(4) Simultaneous resonance $(\omega_1=\omega_2),(\Omega_1=\omega_1),(\Omega_2=\omega_1\pm\Omega_3)$.

Then deduce the worst ones, two of the worst cases have been chosen to study the system stability, and compare the effect of absorber on these cases. The selected resonance cases are $(\omega_1=\omega_2),(\Omega_1=\omega_1),(\Omega_2=\omega_1\pm\Omega_3)$

3. First Primary and Internal resonance

Simultaneous resonance $(\omega_1=\omega_2),(\Omega_1=\omega_1)$ .In this case we introduce the detuning parameter σ_1,σ_2 according to:

$$\Omega_1=\omega_1+\varepsilon\sigma_1 \quad (19)$$

$$\omega_2=\omega_1+\varepsilon\sigma_2 \quad (20)$$

where $\sigma_i\,(i=1,2)$ is the detuning parameter .Also for stability investigation, the analysis is limited to the first approximation . So, our solution is only dependent on T_0 and T_1. Substituting Eqs (19) and (20) into Eqs. (15) and (16) and eliminating the secular terms leads to the solvability conditions

$$(-2i\omega_1 D_1 A_0 - i\omega_1\alpha_3 A_0 + \alpha_4 B_0 e^{i\varepsilon\sigma_2 T_1} + i\omega_2 B_0\alpha_1 e^{i\varepsilon\sigma_2 T_1} + \frac{f_1}{2}e^{i\varepsilon\sigma_1 T_1}) = 0 \quad (21)$$

$$(-2i\omega_2 D_1 B_0 - \alpha_4 B_0 - i\omega_2 B_0\alpha_7 + \alpha_8 B_0 + \delta\omega_1^2 A_0 e^{-i\varepsilon\sigma_2 T_1} + i\omega_1\alpha_3 A_0 e^{-i\varepsilon\sigma_2 T_1}) = 0 \quad (22)$$

To analyze the solution of Eqs (21) and (22), it is convenient to express A in the polar form as:

$$A_0 = \frac{1}{2}a_1(T_1)e^{i\theta_1(T_1)} \quad (23)$$

$$B_0 = \frac{1}{2}a_2(T_1)e^{i\theta_2(T_1)} \quad (24)$$

where $a_i(i=1,2), \theta_i(i=1,2)$ are unknown real-valued function .Inserting equation (23) and (24) into Eqs (21)and (22) Let $(\sigma_2 T_1 - \theta_1 + \theta_2 = \gamma_2), (\sigma_1 T_1 - \theta_1 = \gamma_1)$ and separating the real and imaginary parts we have the following:

$$a_1' = \frac{f_1}{2\omega_1}\sin\gamma_1 - \frac{\alpha_3}{2}a_1 + \frac{\alpha_4}{2\omega_1}a_2\sin\gamma_2 + \frac{\omega_2\alpha_1}{2\omega_1}a_2\cos\gamma_2 \quad (25)$$

$$\theta_1' a_1 = -\frac{f_1}{2\omega_1}\cos\gamma_1 - \frac{\alpha_4}{2\omega_1}a_2\cos\gamma_2 + \frac{\omega_2\alpha_1}{2\omega_1}a_2\sin\gamma_2 \quad (26)$$

$$a_2' = -\frac{\alpha_4}{2}a_2 - \frac{\delta\omega_1^2}{\omega_2}a_1\sin\gamma_2 + \frac{\omega_1\alpha_3}{2\omega_2}a_1\cos\gamma_2 \quad (27)$$

$$\theta_2' a_2 = \frac{(\alpha_4 - \alpha_8)}{2\omega_2}a_2 - \frac{\delta\omega_1^2}{\omega_2}a_1\cos\gamma_2 - \frac{\omega_1\alpha_3}{2\omega_2}a_1\sin\gamma_2 \quad (28)$$

For steady solutions $a_i' = 0, \gamma_i' = 0$ and the periodic solution at the fixed points corresponding to Eqs (25)-(28) is given by:

$$\frac{f_1}{2\omega_1}\sin\gamma_1 = \frac{\alpha_3}{2}a_1 + \frac{\alpha_4}{2\omega_1}a_2\sin\gamma_2 + \frac{\omega_2\alpha_1}{2\omega_1}a_2\cos\gamma_2 \quad (29)$$

$$\sigma_1 a_1 = -\frac{f_1}{2\omega_1}\cos\gamma_1 - \frac{\alpha_4}{2\omega_1}a_2\cos\gamma_2 + \frac{\omega_2\alpha_1}{2\omega_1}a_2\sin\gamma_2 \quad (30)$$

$$\sin\gamma_2 = \left(\frac{M_4}{M_3}\right)\frac{a_2}{a_1} \quad (31)$$

$$\cos\gamma_2 = \frac{\alpha_4\omega_2 a_2}{\omega_1\alpha_3 a_1} + \left(\frac{2\delta\omega_1 M_4}{\alpha_3 M_3}\right)\frac{a_2}{a_1} \quad (32)$$

From Eqs (29)-(32) we get the corresponding frequency response equation (FRE) is:

$$\left(\frac{\alpha_3}{2}a_1-\frac{\alpha_4 M_4}{2\omega_1 M_3}\frac{a_2^2}{a_1}-\frac{\omega_2^2\alpha_1}{2\omega_1^2\alpha_3^2}\frac{a_2^2}{a_1}-\frac{\omega_2\alpha_1\delta M_4}{\alpha_3 M_3}\frac{a_2^2}{a_1}\right)^2+\left(-\sigma_1 a_1+\frac{\omega_2\alpha_1 M_4}{2\omega_1 M_3}\frac{a_2^2}{a_1}-\frac{\omega_2\alpha_4^2}{2\omega_1^2\alpha_3^2}\frac{a_2^2}{a_1}\right.$$
$$\left.-\frac{\alpha_4\delta M_4}{\alpha_3 M_3}\frac{a_2^2}{a_1}\right)^2-\left(\frac{f_1}{2\omega_1}\right)^2=0 \qquad (33)$$

$$a_1^2=\frac{M_2^2}{M_1^2}a_2^2 \qquad (34)$$

where $M_i\left(i=1,2,3,4\right)$ are defined in appendix

3.1.1 Linear solution

To study the stability of the linear solution of the obtained fixed points, let us consider A and B in the forms:

$$A_0(T_1)=\frac{1}{2}(p_1-iq_1)e^{i\delta_1 T_1} \qquad (35)$$

$$B_0(T_1)=\frac{1}{2}(p_2-iq_2)e^{i\delta_2 T_1} \qquad (36)$$

where p_1, q_1, p_2 *and* q_2 are real values and considering $\delta_1=\sigma_1, \delta_2=\sigma_1-\sigma_2$.
Substituting from Eqs (35) and (36) into the linear parts of Eqs (21) and (22) and separating real and imaginary parts, the following system of equations are obtained:

$$p_1'=-\left(-\frac{1}{2}\alpha_3\right)p_1+\left(-\delta_1\right)q_1+\left(\frac{\omega_2\alpha_1}{2\omega_1}\right)p_2+\left(-\frac{\alpha_4}{\omega_1}\right)q_2 \qquad (37)$$

$$q_1'=\left(\delta_1\right)p_1-\left(\frac{1}{2}\alpha_3\right)q_1+\left(\frac{\alpha_4}{\omega_1}\right)p_2+\left(\frac{\omega_2\alpha_1}{2\omega}\right)q_2+\frac{f_1}{2\omega_1} \qquad (38)$$

$$p_2'=\left(\frac{\omega_1\alpha_3}{2\omega_2}\right)p_1+\left(-\frac{\delta\omega_1^2}{2\omega_2}\right)p_1+\left(-\frac{\alpha_4}{2}\right)p_2+\left(-\delta_2+\frac{(\alpha_4-\alpha_8)}{2\omega_2}\right)q_2 \qquad (39)$$

$$q_2'=\left(\frac{\delta\omega_1^2}{2\omega_2}\right)p_1+\left(\frac{\omega_1\alpha_3}{2\omega_2}\right)q_1+\left(\delta_2+\frac{(\alpha_8-\alpha_4)}{2\omega_2}\right)p_2+\left(-\frac{\alpha_4}{2}\right)q_2 \qquad (40)$$

The above equations can be written in a matrix form as:

$$\begin{bmatrix} p_1' \\ q_1' \\ p_2' \\ q_2' \end{bmatrix}=\begin{bmatrix} -\frac{1}{2}\alpha_3 & -\delta_1 & \frac{\omega_2\alpha_1}{2\omega_1} & -\frac{\alpha_4}{\omega_1} \\ \delta_1 & -\frac{1}{2}\alpha_3 & \frac{\alpha_4}{\omega_1} & \frac{\omega_2\alpha_1}{2\omega} \\ \frac{\omega_1\alpha_3}{2\omega_2} & -\frac{\delta\omega_1^2}{2\omega_2} & -\frac{\alpha_4}{2} & \left(-\delta_2+\frac{(\alpha_4-\alpha_8)}{2\omega_2}\right) \\ \frac{\delta\omega_1^2}{2\omega_2} & \frac{\omega_1\alpha_3}{2\omega_2} & \left(\delta_2+\frac{(\alpha_8-\alpha_4)}{2\omega_2}\right) & -\frac{\alpha_4}{2} \end{bmatrix}\begin{bmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{bmatrix} \qquad (41)$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$\begin{vmatrix} \lambda-\left(-\frac{1}{2}\alpha_3\right) & -\delta_1 & \frac{\omega_2\alpha_1}{2\omega_1} & -\frac{\alpha_4}{\omega_1} \\ \delta_1 & \lambda-\left(-\frac{1}{2}\alpha_3\right) & \frac{\alpha_4}{\omega_1} & \frac{\omega_2\alpha_1}{2\omega} \\ \frac{\omega_1\alpha_3}{2\omega_2} & -\frac{\delta\omega_1^2}{2\omega_2} & \lambda-\left(-\frac{\alpha_4}{2}\right) & \left(-\delta_2+\frac{(\alpha_4-\alpha_8)}{2\omega_2}\right) \\ \frac{\delta\omega_1^2}{2\omega_2} & \frac{\omega_1\alpha_3}{2\omega_2} & \left(\delta_2+\frac{(\alpha_8-\alpha_4)}{2\omega_2}\right) & \lambda-\left(-\frac{\alpha_4}{2}\right) \end{vmatrix}=0 \qquad (42)$$

After extract we obtain that:

$$\lambda^4+r_1\lambda^3+r_2\lambda^2+r_3\lambda+r_4=0 \qquad (43)$$

where r_1, r_2, r_3 and r_4 are defined in Appendix.

According to Routh-Huriwitz criterion, the above linear solution is stable if the following inequalities are satisfied:

$$r_1>0,\ r_1 r_2-r_3>0,\ r_3(r_1 r_2-r_3)-r_1^2 r_4>0,\ r_4>0$$

3.1.2Non-linear solution

To determine the stability of the fixed points, one lets

$$a_n=a_{n0}+a_{n1}(n=1,2),\ \gamma_m=\gamma_{m0}+\gamma_{m1}(m=1,2) \qquad (44)$$

Where a_{10}, a_{20} and γ_{m0} are solutions of Eqs (29) - (33) and $a_{11}, a_{21}, \gamma_{m1}$ are perturbations which are assumed to be small compared to a_{10}, a_{20} and γ_{m0}. Substituting Eq (44) into Eqs (25)-(28) using Eqs (29) - (32) and keeping only the linear terms, we obtain:

$$a'_{11}=\left[-\frac{\alpha_3}{2}\right]a_{11}+\left[\frac{f_1}{2\omega_1}\cos\gamma_{10}\right]\gamma_{11}+\left[\frac{\omega_2\alpha_1}{2\omega_1}\cos\gamma_{20}+\frac{\alpha_4}{2\omega_1}\sin\gamma_{20}\right]a_{21}$$
$$+\left[-\frac{\omega_2\alpha_1}{2\omega_1}a_{20}\sin\gamma_{20}+\frac{\alpha_4}{2\omega_1}a_{20}\cos\gamma_{20}\right]\gamma_{21} \qquad (45)$$

$$\gamma'_{11}=\left[\frac{\sigma_1}{a_{10}}-\right]a_{11}-\left[\frac{f_1}{2\omega_1 a_{10}}\sin\gamma_{10}\right]\gamma_{11}+\left[-\frac{\omega_2\alpha_1}{2\omega_1 a_{10}}\sin\gamma_{20}+\frac{\alpha_4}{2\omega_1 a_{10}}\cos\gamma_{20}\right]a_{21}$$
$$+\left[-\frac{\omega_2\alpha_1 a_{20}}{2\omega_1 a_{10}}\cos\gamma_{20}-\frac{\alpha_4 a_{20}}{2\omega_1 a_{10}}\sin\gamma_{20}\right]\gamma_{21} \qquad (46)$$

$$a'_{21}=\left[-\frac{\omega_1^2\delta}{\omega_2}\sin\gamma_{20}+\frac{\omega_1\alpha_3}{2\omega_2}\cos\gamma_{20}\right]a_{11}+\left[-\frac{\alpha_4}{2}\right]a_{21}+\left[-\frac{\omega_1^2\delta a_{10}}{\omega_2}\cos\gamma_{20}-\frac{\omega_1\alpha_3 a_{10}}{2\omega_2}\sin\gamma_{20}\right]\gamma_{21} \qquad (47)$$

$$\gamma'_{21}=\left[-\frac{\omega_1^2\delta}{\omega_2 a_{20}}\cos\gamma_{20}-\frac{\omega_1\alpha_3}{2\omega_2 a_{20}}\sin\gamma_{20}+\frac{\sigma_1}{a_{10}}\right]a_{11}+\left[-\frac{f_1}{2\omega_1 a_{10}}\sin\gamma_{10}\right]\gamma_{11}+\left[-\frac{(\sigma_1-\sigma_2)}{2}+\frac{(\alpha_4-\alpha_8)}{2\omega_2 a_{20}}\right.$$
$$\left.+\frac{\alpha_4}{2\omega_1 a_{10}}\cos\gamma_{20}-\frac{\omega_2\alpha_1}{2\omega_1 a_{10}}\sin\gamma_{20}\right]a_{21}+\left[-\frac{\omega_1^2\delta a_{10}}{\omega_2 a_{20}}\sin\gamma_{20}-\frac{\omega_1\alpha_3 a_{10}}{2\omega_2 a_{20}}\cos\gamma_{20}-\frac{\omega_2\alpha_1 a_{20}}{2\omega_1 a_{10}}\cos\gamma_{20}-\frac{\alpha_4 a_{20}}{2\omega_1 a_{10}}\sin\gamma_{20}\right]\gamma_{21}$$
$$(48)$$

3.2 Second combined and internal resonance

Simultaneous resonance $\left(\omega_1=\omega_2\right),\left(\Omega_2=\omega_1+\Omega_3\right)$.In this case we introduce the detuning parameter σ_1,σ_2 according to:

$$\Omega_1=\omega_1+\Omega_3+\varepsilon\sigma_1 \tag{49}$$

$$\omega_2=\omega_1+\varepsilon\sigma_2 \tag{50}$$

where $\sigma_i\left(i=1,2\right)$ is the detuning parameter .Also for stability investigation, the analysis is limited to the first approximation . So, our solution is only dependent on T_0 and T_1. Substituting Eqs (49) and (50) into Eqs. (15) and (16) and eliminating the secular terms leads to the solvability conditions

$$(-2i\omega_1 D_1 A_0-i\omega_1\alpha_3 A_0+\alpha_4 B_0 e^{i\varepsilon\sigma_2 T_1}+i\omega_2 B_0\alpha_1 e^{i\varepsilon\sigma_2 T_1}+\frac{if_2}{4}e^{i\varepsilon\sigma_1 T_1})=0 \tag{51}$$

$$(-2i\omega_2 D_1 B_0-\alpha_4 B_0-i\omega_2 B_0\alpha_7+\alpha_8 B_0+\delta\omega_1^2 A_0 e^{-i\varepsilon\sigma_2 T_1}+i\omega_1\alpha_3 A_0 e^{-i\varepsilon\sigma_2 T_1})=0 \tag{52}$$

To analyze the solution of Eqs (51) and (52), it is convenient to express A in the polar form as:

$$A_0=\frac{1}{2}a_1(T_1)e^{i\theta_1(T_1)} \tag{53}$$

$$B_0=\frac{1}{2}a_2(T_1)e^{i\theta_2(T_1)} \tag{54}$$

where $a_i(i=1,2),\theta_i(i=1,2)$ are unknown real-valued function .Inserting equation (53) and (54) into Eqs (51)and (52) Let $(\sigma_2 T_1-\theta_1+\theta_2=\gamma_2),(\sigma_1 T_1-\theta_1=\gamma_1)$ and separating the real and imaginary parts we have the following:

$$a_1'=\frac{f_2}{4\omega_1}\cos\gamma_1-\frac{\alpha_3}{2}a_1+\frac{\alpha_4}{2\omega_1}a_2\sin\gamma_2+\frac{\omega_2\alpha_1}{2\omega_1}a_2\cos\gamma_2 \tag{55}$$

$$\theta_1' a_1=\frac{f_2}{4\omega_1}\sin\gamma_1-\frac{\alpha_4}{2\omega_1}a_2\cos\gamma_2+\frac{\omega_2\alpha_1}{2\omega_1}a_2\sin\gamma_2 \tag{56}$$

$$a_2'=-\frac{\alpha_4}{2}a_2-\frac{\delta\omega_1^2}{\omega_2}a_1\sin\gamma_2+\frac{\omega_1\alpha_3}{2\omega_2}a_1\cos\gamma_2 \tag{57}$$

$$\theta_2' a_2=\frac{\left(\alpha_4-\alpha_8\right)}{2\omega_2}a_2-\frac{\delta\omega_1^2}{\omega_2}a_1\cos\gamma_2-\frac{\omega_1\alpha_3}{2\omega_2}a_1\sin\gamma_2 \tag{58}$$

For steady solutions $a_i'=0,\gamma_i'=0$ and the periodic solution at the fixed points corresponding to Eqs (55)-(58) is given by:

$$\frac{f_2}{4\omega_1}\cos\gamma_1=\frac{\alpha_3}{2}a_1-\frac{\alpha_4}{2\omega_1}a_2\sin\gamma_2-\frac{\omega_2\alpha_1}{2\omega_1}a_2\cos\gamma_2 \tag{59}$$

$$\sigma_1 a_1 = \frac{f_2}{4\omega_1}\cos\gamma_1 - \frac{\alpha_4}{2\omega_1}a_2\cos\gamma_2 + \frac{\omega_2\alpha_1}{2\omega_1}a_2\sin\gamma_2 \tag{60}$$

$$\sin\gamma_2 = \left(\frac{M_4}{M_3}\right)\frac{a_2}{a_1} \tag{61}$$

$$\cos\gamma_2 = \frac{\alpha_4\omega_2 a_2}{\omega_1\alpha_3 a_1} + \left(\frac{2\delta\omega_1 M_4}{\alpha_3 M_3}\right)\frac{a_2}{a_1} \tag{62}$$

From Eqs (59)-(62) we get the corresponding frequency response equation (FRE) is:

$$\left(\frac{\alpha_3}{2}a_1 - \frac{\alpha_4 M_4}{2\omega_1 M_3}\frac{a_2^2}{a_1} - \frac{\omega_2^2\alpha_1}{2\omega_1^2\alpha_3^2}\frac{a_2^2}{a_1} - \frac{\omega_2\alpha_1\delta M_4}{\alpha_3 M_3}\frac{a_2^2}{a_1}\right)^2 + \left(\sigma_1 a_1 - \frac{\omega_2\alpha_1 M_4}{2\omega_1 M_3}\frac{a_2^2}{a_1} + \frac{\omega_2\alpha_4^2}{2\omega_1^2\alpha_3^2}\frac{a_2^2}{a_1}\right.$$

$$\left. + \frac{\alpha_4\delta M_4}{\alpha_3 M_3}\frac{a_2^2}{a_1}\right)^2 - \left(\frac{f_2}{4\omega_1}\right)^2 = 0 \tag{63}$$

$$a_1^2 = \frac{M_2^2}{M_1^2}a_2^2 \tag{64}$$

where $M_i\,(i = 1,2,3,4)$ are defined in appendix

3.2.1 Linear solution

To study the stability of the linear solution of the obtained fixed points, let us consider A and B in the forms:

$$A_0(T_1) = \frac{1}{2}(p_1 - iq_1)e^{i\delta_1 T_1} \tag{65}$$

$$B_0(T_1) = \frac{1}{2}(p_2 - iq_2)e^{i\delta_2 T_1} \tag{66}$$

where p_1, q_1, p_2 *and* q_2 are real values and considering $\delta_1 = \sigma_1, \delta_2 = \sigma_1 - \sigma_2$.

Substituting from Eqs (65) and (66) into the linear parts of Eqs (51) and (52) and separating real and imaginary parts, the following system of equations are obtained:

$$p_1' = -\left(-\frac{1}{2}\alpha_3\right)p_1 + (-\delta_1)q_1 + \left(\frac{\omega_2\alpha_1}{2\omega_1}\right)p_2 + \left(-\frac{\alpha_4}{\omega_1}\right)q_2 + \frac{f_2}{4\omega_2} \tag{67}$$

$$q_1' = (\delta_1)p_1 - \left(\frac{1}{2}\alpha_3\right)q_1 + \left(\frac{\alpha_4}{\omega_1}\right)p_2 + \left(\frac{\omega_2\alpha_1}{2\omega}\right)q_2 \tag{68}$$

$$p_2' = \left(\frac{\omega_1\alpha_3}{2\omega_2}\right)p_1 + \left(-\frac{\delta\omega_1^2}{2\omega_2}\right)p_1 + \left(-\frac{\alpha_4}{2}\right)p_2 + \left(-\delta_2 + \frac{(\alpha_4 - \alpha_8)}{2\omega_2}\right)q_2 \tag{69}$$

$$q_2' = \left(\frac{\delta\omega_1^2}{2\omega_2}\right)p_1 + \left(\frac{\omega_1\alpha_3}{2\omega_2}\right)q_1 + \left(\delta_2 + \frac{(\alpha_8 - \alpha_4)}{2\omega_2}\right)p_2 + \left(-\frac{\alpha_4}{2}\right)q_2 \tag{70}$$

The above equations can be written in a matrix form as:

$$\begin{bmatrix} p_1' \\ q_1' \\ p_2' \\ q_2' \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\alpha_3 & -\delta_1 & \frac{\omega_2\alpha_1}{2\omega_1} & -\frac{\alpha_4}{\omega_1} \\ \delta_1 & -\frac{1}{2}\alpha_3 & \frac{\alpha_4}{\omega_1} & \frac{\omega_2\alpha_1}{2\omega} \\ \frac{\omega_1\alpha_3}{2\omega_2} & -\frac{\delta\omega_1^2}{2\omega_2} & -\frac{\alpha_4}{2} & \left(-\delta_2 + \frac{(\alpha_4-\alpha_8)}{2\omega_2}\right) \\ \frac{\delta\omega_1^2}{2\omega_2} & \frac{\omega_1\alpha_3}{2\omega_2} & \left(\delta_2 + \frac{(\alpha_8-\alpha_4)}{2\omega_2}\right) & -\frac{\alpha_4}{2} \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{bmatrix} \tag{71}$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$\begin{vmatrix} \lambda-\left(-\frac{1}{2}\alpha_3\right) & -\delta_1 & \frac{\omega_2\alpha_1}{2\omega_1} & -\frac{\alpha_4}{\omega_1} \\ \delta_1 & \lambda-\left(-\frac{1}{2}\alpha_3\right) & \frac{\alpha_4}{\omega_1} & \frac{\omega_2\alpha_1}{2\omega} \\ \frac{\omega_1\alpha_3}{2\omega_2} & -\frac{\delta\omega_1^2}{2\omega_2} & \lambda-\left(-\frac{\alpha_4}{2}\right) & \left(-\delta_2 + \frac{(\alpha_4-\alpha_8)}{2\omega_2}\right) \\ \frac{\delta\omega_1^2}{2\omega_2} & \frac{\omega_1\alpha_3}{2\omega_2} & \left(\delta_2 + \frac{(\alpha_8-\alpha_4)}{2\omega_2}\right) & \lambda-\left(-\frac{\alpha_4}{2}\right) \end{vmatrix} = 0 \tag{72}$$

After extract we obtain that:

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0 \tag{73}$$

where r_1, r_2, r_3 and r_4 are defined in Appendix.

According to Routh-Huriwitz criterion, the above linear solution is stable if the following inequalities are satisfied:

$$r_1 > 0, r_1 r_2 - r_3 > 0, r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0, r_4 > 0$$

3.2.2 Non-linear solution

To determine the stability of the fixed points, one lets

$$a_n = a_{n0} + a_{n1}(n=1,2), \gamma_m = \gamma_{m0} + \gamma_{m1}\left(m=1,2\right) \tag{74}$$

Where a_{10}, a_{20} and γ_{m0} are solutions of Eqs (59) - (62) and $a_{11}, a_{21}, \gamma_{m1}$ are perturbations which are assumed to be small compared to a_{10}, a_{20} and γ_{m0}. Substituting Eq (48) into Eqs (55)-(58) using Eqs (59)- (62) and keeping only the linear terms, we obtain:

$$a_{11}' = \left[-\frac{\alpha_3}{2}\right]a_{11} + \left[-\frac{f_2}{4\omega_1}\sin\gamma_{10}\right]\gamma_{11} + \left[\frac{\omega_2\alpha_1}{2\omega_1}\cos\gamma_{20} + \frac{\alpha_4}{2\omega_1}\sin\gamma_{20}\right]a_{21}$$
$$+\left[-\frac{\omega_2\alpha_1}{2\omega_1}a_{20}\sin\gamma_{20} + \frac{\alpha_4}{2\omega_1}a_{20}\cos\gamma_{20}\right]\gamma_{21} \tag{75}$$

$$\gamma_{11}' = \left[\frac{\sigma_1}{a_{10}}\right]a_{11} - \left[\frac{f_2}{4\omega_1 a_{10}}\cos\gamma_{10}\right]\gamma_{11} + \left[-\frac{\omega_2\alpha_1}{2\omega_1 a_{10}}\sin\gamma_{20} + \frac{\alpha_4}{2\omega_1 a_{10}}\cos\gamma_{20}\right]a_{21}$$
$$+\left[-\frac{\omega_2\alpha_1 a_{20}}{2\omega_1 a_{10}}\cos\gamma_{20} - \frac{\alpha_4 a_{20}}{2\omega_1 a_{10}}\sin\gamma_{20}\right]\gamma_{21} \tag{76}$$

$$a'_{21}=\left[-\frac{\omega_1^2\delta}{\omega_2}\sin\gamma_{20}+\frac{\omega_1\alpha_3}{2\omega_2}\cos\gamma_{20}\right]a_{11}+\left[-\frac{\alpha_4}{2}\right]a_{21}+\left[-\frac{\omega_1^2\delta a_{10}}{\omega_2}\cos\gamma_{20}-\frac{\omega_1\alpha_3 a_{10}}{2\omega_2}\sin\gamma_{20}\right]\gamma_{21} \quad (77)$$

$$\gamma'_{21}=\left[-\frac{\omega_1^2\delta}{\omega_2 a_{20}}\cos\gamma_{20}-\frac{\omega_1\alpha_3}{2\omega_2 a_{20}}\sin\gamma_{20}+\frac{\sigma_1}{a_{10}}\right]a_{11}+\left[-\frac{f_2}{4\omega_1 a_{10}}\cos\gamma_{10}\right]\gamma_{11}+\left[-\frac{(\sigma_1-\sigma_2)}{2}+\frac{(\alpha_4-\alpha_8)}{2\omega_2 a_{20}}\right.$$
$$\left.+\frac{\alpha_4}{2\omega_1 a_{10}}\cos\gamma_{20}-\frac{\omega_2\alpha_1}{2\omega_1 a_{10}}\sin\gamma_{20}\right]a_{21}+\left[-\frac{\omega_1^2\delta a_{10}}{\omega_2 a_{20}}\sin\gamma_{20}-\frac{\omega_1\alpha_3 a_{10}}{2\omega_2 a_{20}}\cos\gamma_{20}-\frac{\omega_2\alpha_1 a_{20}}{2\omega_1 a_{10}}\cos\gamma_{20}-\frac{\alpha_4 a_{20}}{2\omega_1 a_{10}}\sin\gamma_{20}\right]\gamma_{21}$$
$$(78)$$

4. Results and discussion

To study behavior of the main system numerically the (Rung-Kutta method) of the nonlinear system, given by Eqs (3)and (4) at basic without absorber, the Primary resonance case $(\Omega_1=\omega_1)$ is obtained as shown in figures (2)-(7).these solutions are obtained at selected values $(\Omega_1=1, \Omega_1=\omega_1, \omega_2=\omega_1)$and $(\Omega_2=\omega_1+\Omega_3), (\omega_1=\omega_2)$.

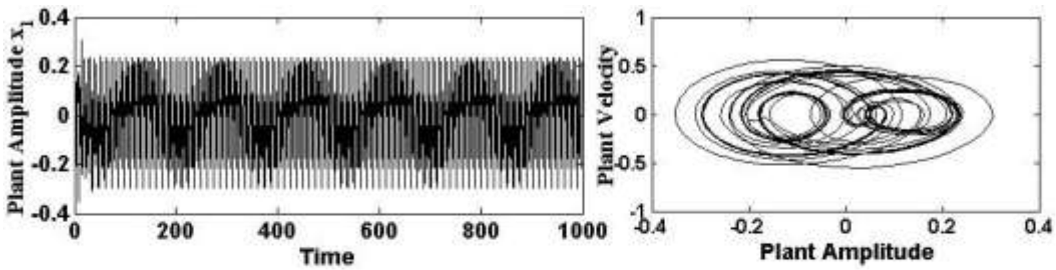


Figure2 Response of the system without absorber at basic case

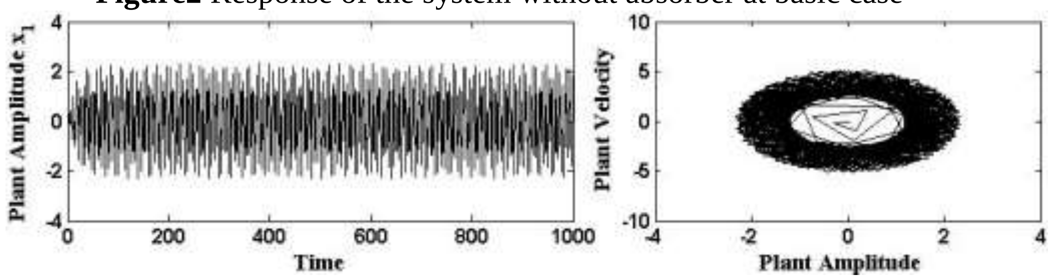


Figure3 Response of the system at resonance case $(\Omega_1=\omega_1=\omega_2)$

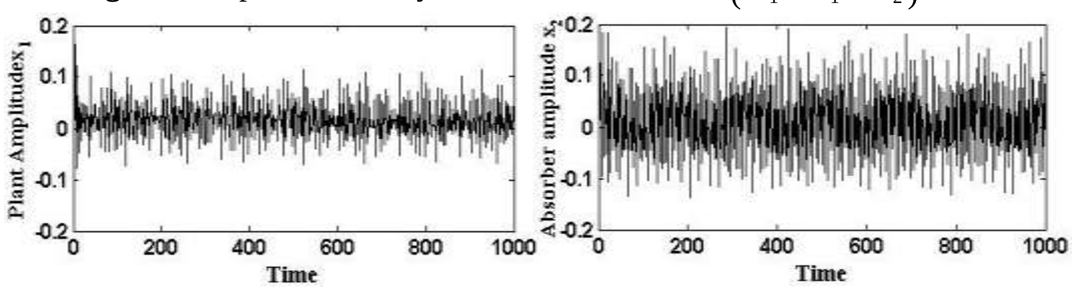


Figure4 Response of the system with absorber in resonance case $(\Omega_1=\omega_1=\omega_2)$

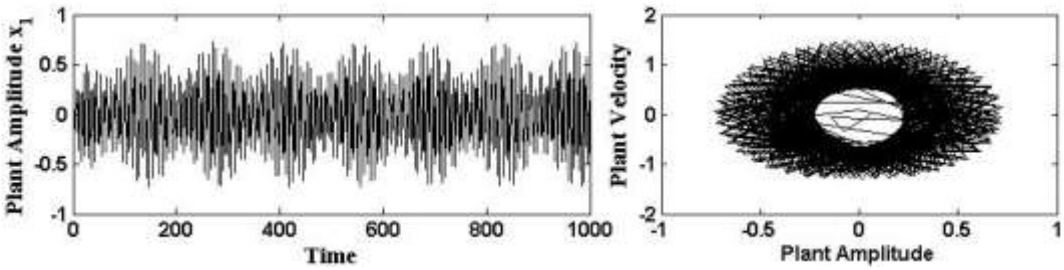


Figure5 Response of the system at resonance case $(\Omega_2 = \omega_1 + \Omega_3, \omega_1 = \omega_2)$

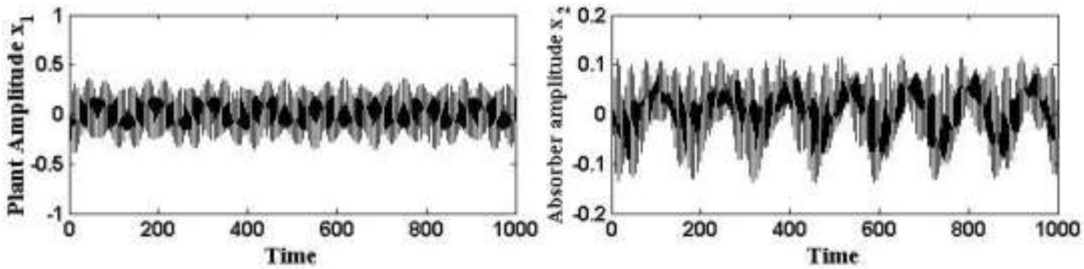


Figure6 Response of the system with absorber in resonance case $(\Omega_2 = \omega_1 + \Omega_3, \omega_1 = \omega_2)$

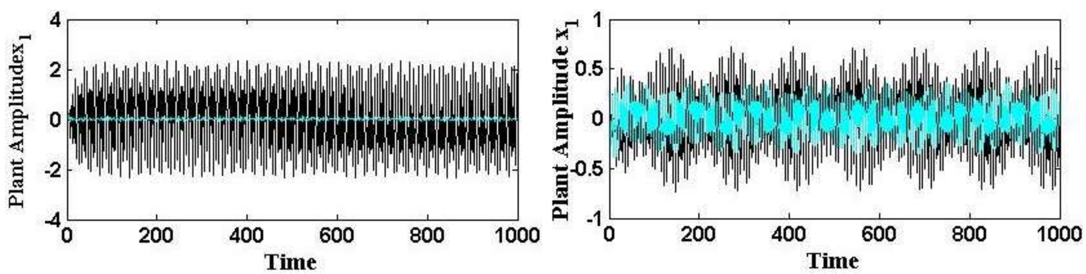


Figure7 Comparison between Responses of the system under effect of absorber in resonance cases

Fig. (2) Show that study of amplitude on the main system in the basic case without absorber of the selection of values as ($f_1 = 0.8$, $f_2 = 0.4$, $\alpha_1 = 0.08, \alpha_2 = 0.8, \alpha_3 = 0.164, \alpha_4 = 12$, $\alpha_5 = 0.003, \alpha_6 = 12, \varepsilon = 0.3$, $\omega_1 = 2.1869$, $\Omega_2 = 3, \Omega_3 = 0.5$, $\Omega_1 = 1$),the amplitude in the system in the basic without absorber Show that effect of passive control where the stability is reached (approximately 0.2) Fig. (3) We study in this figure the amplitude in the system at the response case. First, at (we study Trivial and internal response case $(\Omega_1 = \omega_1, \omega_1 = \omega_2)$), then we find that in this case the amplitude at maximum reached (up to approximately 2), Fig. (4) Show the effect of passive control at the response case at $(\Omega_1 = \omega_1, \omega_1 = \omega_2)$ we find absorber able to reduce and control vibration significantly until amplitude reached (up to approximately 0.1). Fig. (5) We study in this figure the amplitude in the system at the response case. Second, at (we study combined and internal response case $(\Omega_2 = \omega_1 + \Omega_3, \omega_1 = \omega_2)$), then we find that in this case the amplitude at maximum reached (up to approximately 0.8), Fig. (6) Show the effect of passive control at the response case at $(\Omega_2 = \omega_1 + \Omega_3, \omega_1 = \omega_2)$ we find absorber able to reduce and control vibration significantly until amplitude reached (up to approximately 0.3). Fig. (7) A comparison between the effect of the absorber on the two resonance states, and the effect of the first case is greater than the second.

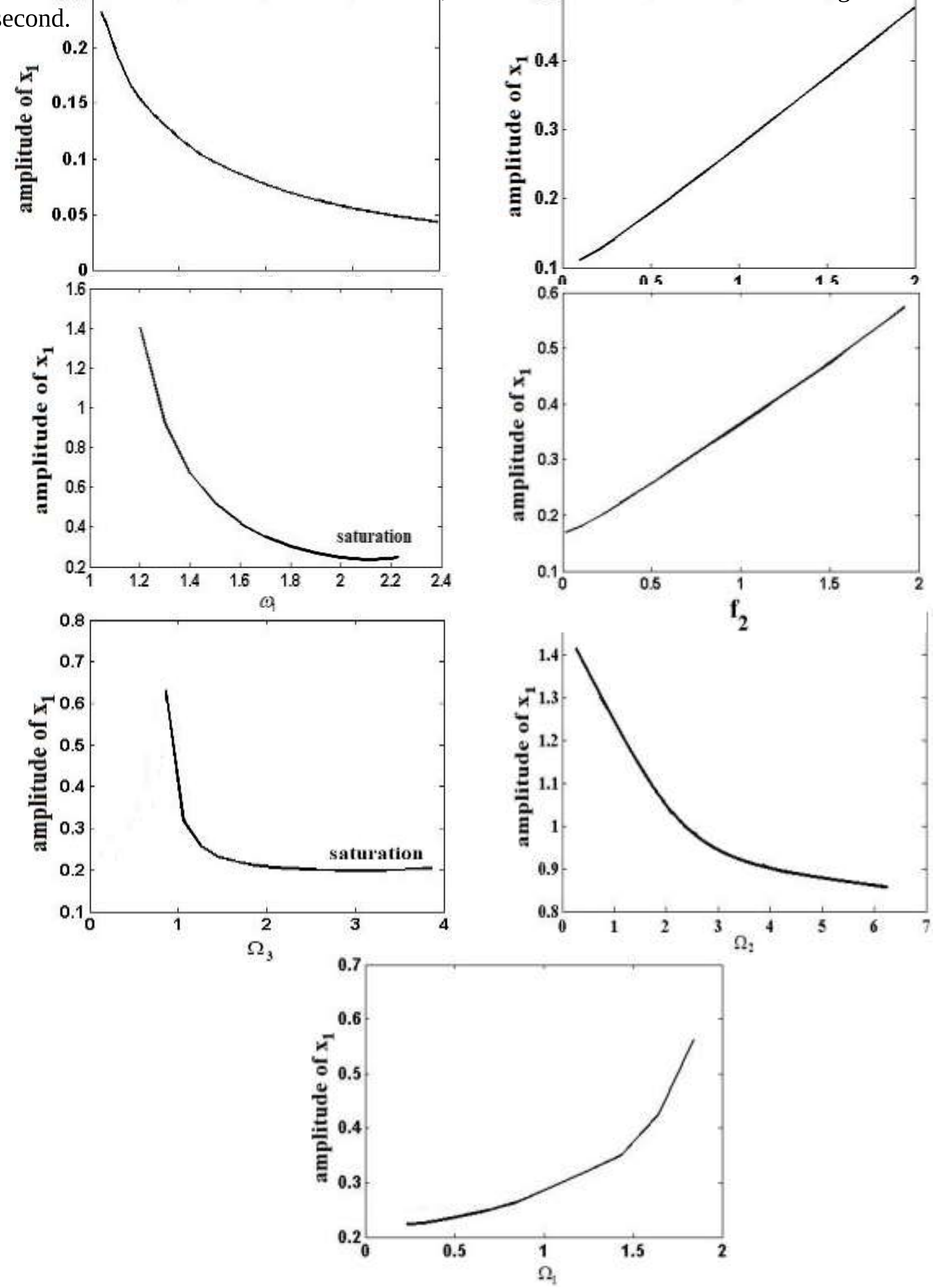
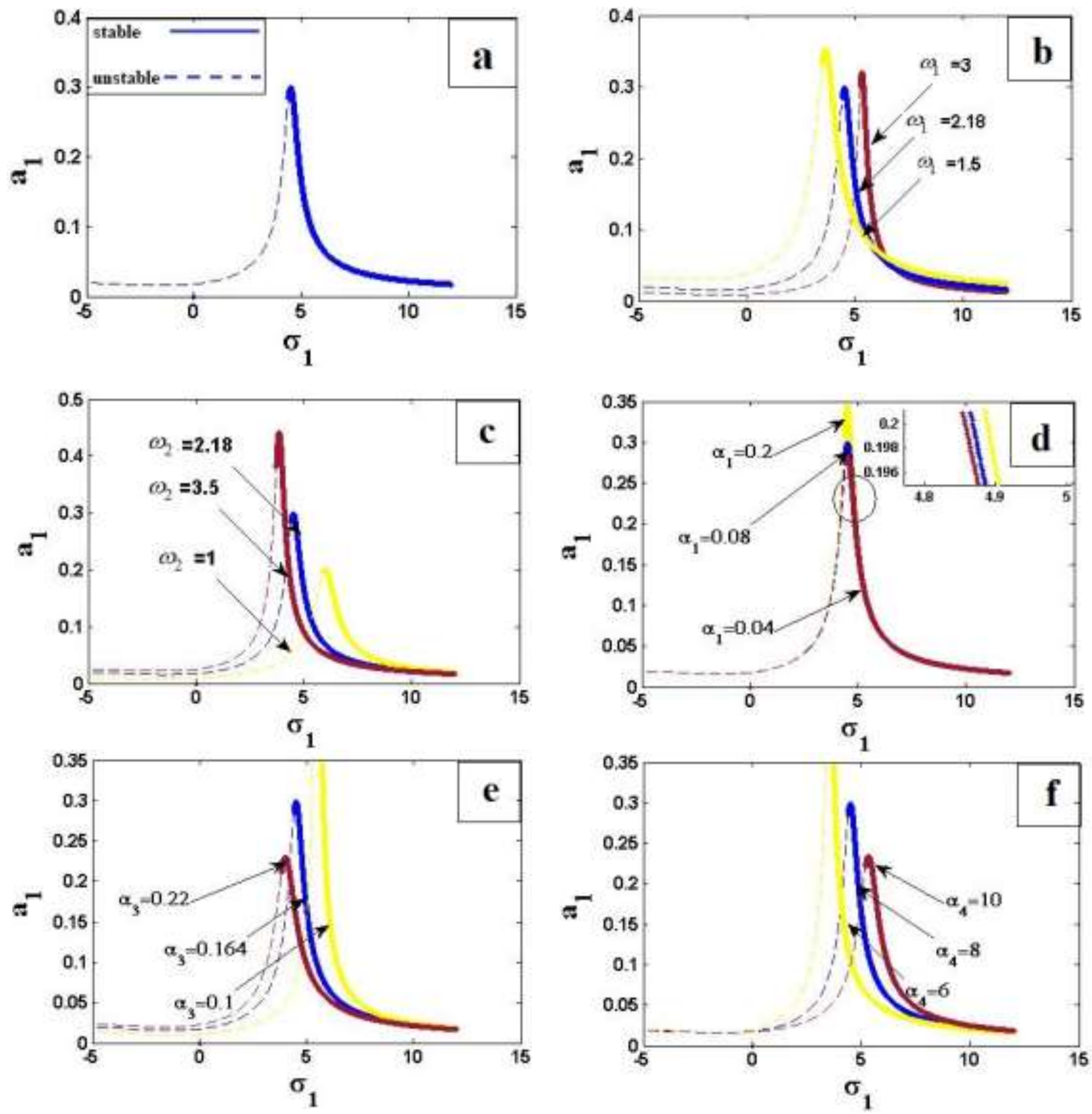


Figure8 Effect of different parameters on the amplitude of the system at the basic case without control

Fig.8 show effect of different parameter on the main system without absorber .can see amplitude increasing as f_1, f_2, Ω_1 is increased .Also when increasing the values, $\omega_1, \Omega_2, \Omega_3$ and α_3 are decreasing as shown.

5. Theoretical frequency and force response curve

The frequency equation is represented graphically by using the numerical methods. The frequency response equation is nonlinear algebraic equation, which are solved numerically by using Newton Raphson method .frequency response equation (21) and (21) is nonlinear algebraic equation, the results are shown in figure (9) for the steady state amplitudes a_1 against parameter σ_1 and .frequency response equation (51) and (51) is nonlinear algebraic equation, the results are shown in figure (10) for the steady state amplitudes a_1 against parameter σ_1 and



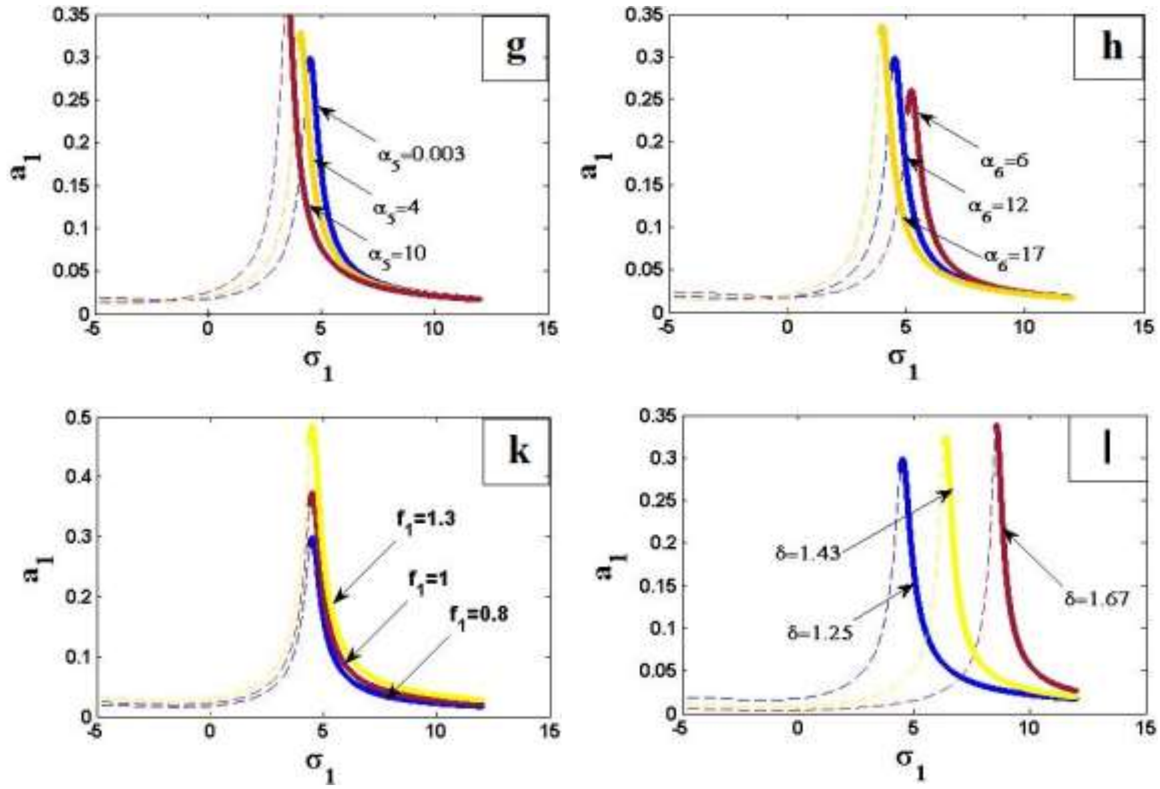
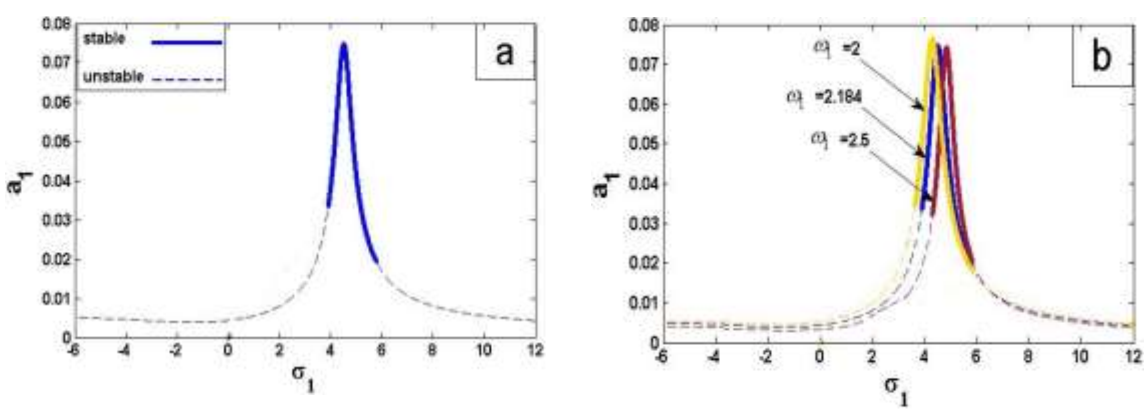


Fig.9 Stability of the practical case (in the first resonance case), $a_2 \neq 0$ on steady state amplitude a_1 against σ_1

Fig.9 show effect of parameter on the system at the steady-state amplitude a_1 decreased when $\alpha_3, \alpha_4, \omega_1$ increasing and amplitude value increased when $f_1, \alpha_1, \alpha_5, \alpha_6, \delta$, Also when α_5, α_6 and ω_2 increasing curve is bending to left, and when ω_1, α_4 and δ increasing curve is bending to right and jump to up when f_1 increasing, we find in(d) the small increase ratio we maximized overlap to show the increase in curves . In the curve (a) shows the relation a_1 and σ_1 it shows that the sable in the right branch and unstable in the left branch, the period of stability and unstability changes with the study of parameters as is shown



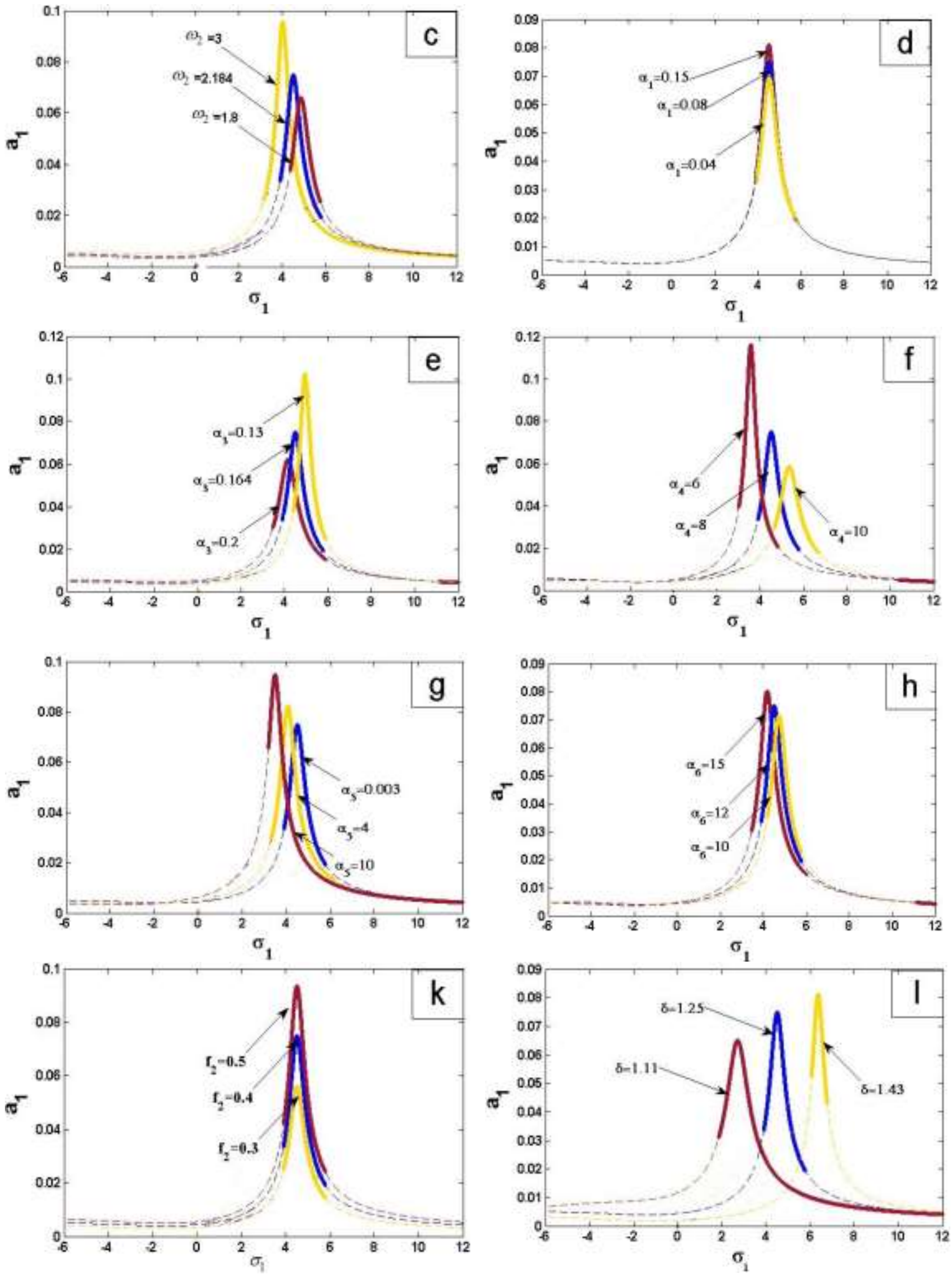


Fig.10 Stability of the practical case (in the second resonance case), $a_2 \neq 0$ on steady state amplitude a_1 against σ_1

Fig.10 show effect of parameter on system at the steady-state amplitude a_1 decreased when $\alpha_3, \alpha_4, \omega_1$ increasing and amplitude value increased when $f_2, \alpha_1, \alpha_5, \alpha_6, \delta$ also when f_2, α_3 increasing curve is bending to up and when $\omega_2, \alpha_3, \alpha_5, \alpha_6$ and δ increasing curve is bending to left also bending to right when increasing ω_1, α_4 . By studying the effect of parameters we found periods of stabilization zone increasing by increasing $\omega_2, \alpha_1, \alpha_5, \alpha_6$, as it is decreasing by an increase δ . In the curve (a) shows the relation a_1 and σ_1 it shows that the sable in the up branch on the curved and unstable in the bottom branch, the period of stability and unstability changes with the study of parameters as is shown

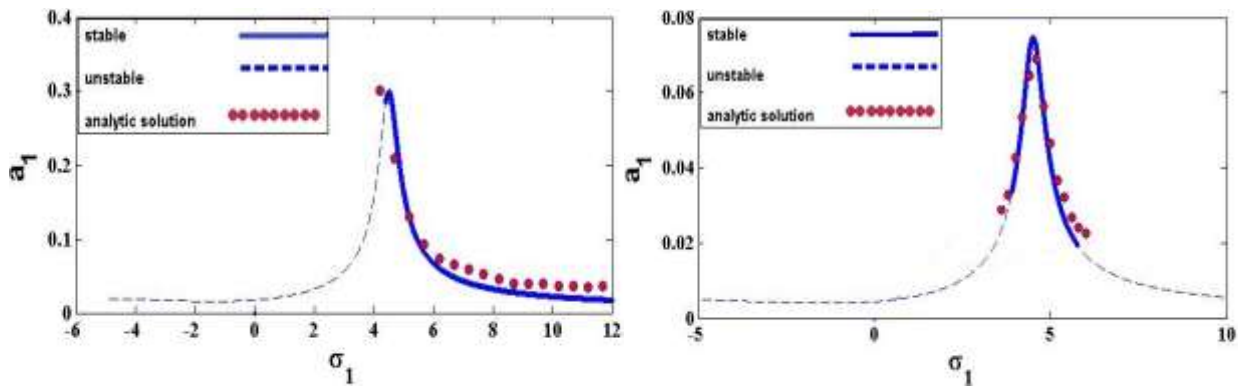


Fig.11 comparison between analytic and approximate solution

7. Conclusions

studying the vibration numerically on the system with and without control .To study the stability of the system obtained numerical solution is investigated using both phase plane methods and frequency response equation in conjunction with study of resonance case, both frequency response equation and results based on the present investigation the above study the following conclusions are shown:

1- The study of resonance cases numerically we conclude that the worst cases primary, combined and internal $(\Omega_1 = \omega_1), (\Omega_2 = \omega_1 + \Omega_3), (\omega_1 = \omega_2)$ on the system with absorber and without and comparison of the absorber effect on both cases.

2- The behavior of amplitude on the worst case with control is more stable and control vibration is almost nonexistent than system without control, where the effect of absorber reduce the amplitude from (2 nearly) to (0.1 nearly) in first case $(\Omega_1 = \omega_1), (\omega_1 = \omega_2)$, and reduce the amplitude from (0.8 nearly) to (0.3 nearly) in the second case $(\Omega_2 = \omega_1 + \Omega_3), (\omega_1 = \omega_2)$

3- The steady state amplitude in the first case of the main system is monotonic increasing function $f_1, \alpha_1, \alpha_5, \alpha_6, \delta$ be on steady state amplitude a_1 against σ_1 ,in the second case monotonic increasing function $f_2, \alpha_1, \alpha_5, \alpha_6, \delta$,on steady state amplitude a_1 against σ_1 .

4- The steady state amplitude in the first case of the main system is monotonic decreasing function $\alpha_3, \alpha_4, \omega_1$ on steady state amplitude a_1 against σ_1, in the second case monotonic decreasing function $\alpha_3, \alpha_4, \omega_1$ on steady state amplitude a_1 against σ_1 .

5- The relation a_1 and σ_1 it shows that the sable in the right branch and un-stable in the left branch, the period of stability and un-stability changes with the study of parameters and relation a_1 and σ_1 in the second case it shows that the sable in the up branch on the curved and unstable in the bottom branch, the period of stability and unstability changes with the study of parameters. And the effect of the first case is greater than the second

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Appendix

$$\omega_1^2=\frac{k_1}{m_1},\omega_2^2=\frac{k_2}{m_2}\ \alpha_1=\frac{c_2}{m_1},\alpha_2=\frac{c_2}{m_2},\alpha_3=\frac{c_1}{m_1}\ ,\ \alpha_4=\frac{k_2}{m_1},\alpha_5=\frac{k_3}{m_1},\alpha_6=\frac{k_3}{m_2}$$

$$\delta=\frac{1}{\varepsilon},\alpha_7=\alpha_1+\alpha_2,\alpha_8=\alpha_5+\alpha_6$$

$$\Gamma_1=\left(\frac{\alpha_4+i\omega_2\alpha_1}{\left(\omega_1^2-\omega_2^2\right)}\right),\Gamma_2=\left(-\frac{\alpha_5}{\left(\omega_1^2-4\omega_2^2\right)}\right),\Gamma_3=\left(\frac{f_1}{2\left(\omega_1^2-\Omega_1^2\right)}\right),\Gamma_4=\left(-\frac{if_2}{4\left(\omega_1^2-\left(\Omega_2+\Omega_3\right)^2\right)}\right)$$

$$\Gamma_5=\left(\frac{if_2}{4\left(\omega_1^2-\left(\Omega_2-\Omega_3\right)^2\right)}\right),\Gamma_6=\left(\frac{\delta\omega_1^2+i\alpha_3\omega_1}{\left(\omega_2^2-\omega_1^2\right)}\right),\Gamma_7=\left(-\frac{f_1}{2\left(\omega_2^2-\Omega_1^2\right)}\right)$$

$$\Gamma_8=\left(\frac{if_2}{4\left(\omega_2^2-\left(\Omega_2+\Omega_3\right)^2\right)}\right),\Gamma_9=\left(-\frac{f_2}{4\left(\omega_2^2-\left(\Omega_2-\Omega_3\right)^2\right)}\right)$$

$$M_1=\left(\left(\frac{\omega_1\alpha_3}{2\omega_2}\right)^2+\left(\frac{\delta\omega_1^2}{\omega_2}\right)^2\right),M_2=\left(\left(\frac{\alpha_4}{2}\right)^2+\left(\frac{\alpha_4-\alpha_8}{2\omega_2}-\left(\sigma_1-\sigma_2\right)\right)^2\right)$$

$$M_3=\left(\frac{\omega_1\alpha_3}{2\omega_2}+\frac{2\omega_1^3\delta}{\alpha_3\omega_2}\right),M_4=\left(\frac{\alpha_4-\alpha_8}{2\omega_2}-\frac{\omega_1\alpha_4\delta}{\alpha_3}-\left(\sigma_1-\sigma_2\right)\right)$$

$$r_1 = (\alpha_4 + \alpha_3), r_2 = \left(\frac{\alpha_4^2 + \alpha_3^2}{4} - \left(\frac{\alpha_8 - \alpha_4}{2\omega_2} + \delta_2 \right)^2 + \alpha_3 \alpha_4 - \delta_1^2 \right)$$

$$r_3 = \left(\frac{\alpha_3 \alpha_4^2 + \alpha_4 \alpha_3^2}{4} - \alpha_4 \delta_1^2 - \alpha_3 \left(\frac{\alpha_8 - \alpha_4}{2\omega_2} + \delta_2 \right)^2 + \frac{\omega_1 \alpha_1 \delta \delta_1}{4} + \frac{\alpha_3 \alpha_4 \delta_1}{2\omega_2} \right)$$

$$r_4 = \left(-\frac{\alpha_3^2}{4} \left(\frac{\alpha_8 - \alpha_4}{2\omega_2} + \delta_2 \right)^2 + \left(\frac{\alpha_4^2 \alpha_3^2}{16} \right) - \left(\frac{\alpha_4 \delta_1}{2} \right)^2 + \delta_1^2 \left(\frac{\alpha_8 - \alpha_4}{2\omega_2} + \delta_2 \right)^2 - \left(-\frac{\delta \delta_1 \omega_1 \alpha_4 \alpha_1}{4} \right) \right.$$

$$\left. + \left(\frac{\alpha_8 - \alpha_4}{2\omega_2} + \delta_2 \right) \left(\frac{\alpha_3 \alpha_1 \delta_1}{4} + \frac{\delta \delta_1 \omega_1 \alpha_4}{2\omega_2} \right) + \left(-\frac{\alpha_4^2 \alpha_3 \delta_1}{2\omega_2} \right) \right)$$