An iterative goal to solve bi-level fractional integer programming problem using fuzzyapproach

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Abstract

This paper solves the bi-level integer linear fractional programming problems based on fuzzy goal approach. At the first phase of the solution algorithm and to avoid the complexity of non-convexity of this problem, the authors finding the convex hull of its original set of constraints using the cutting-plane algorithm, and then the. It makes an extension work of Moitra and Pal (2002) and Pal et al. (2003). In the proposed procedure, the membership functions for the defined fuzzy goals of the decision makers (DMs) objective functions at both levels as well as the membership functions for vector of fuzzy goals of the decision variables controlled by first-level decision maker are developed first in the model formulation of the problem. Then a fuzzy goal programming model to minimize the group regret of degree of satisfactions of both the decision makers is developed to achieve the highest degree (unity) of each of the defined membership function goals to the extent possible by minimizing their deviational variables and thereby obtaining the most satisfactory solution for both decision makers. The method of variable change on the under- and over-attainment variables of the membership goals associated with the fuzzy goals of the model is introduced to solve the problem efficiently by using linear goal programming (LGP) methodology. Illustrative numerical example is given to demonstrate the procedure.

Keywords: *Bi-level optimization; Fraction programming; Integer programming; Goal programming.*

1: Introduction

Bi-level programming (BLP) is a subset of the multi-level programming problem which identified as a mathematical programming problem that solves decentralized planning problems with two decision makers (DMs) in a two- level or hierarchical organization.

Linear Fractional programming problem is that in which maximizes or minimizes objective function is the ratio of numerator and denominator. A usual linear fractional programming problem is a special case of a non-linear programming problem, but it can be transformed into a linear programming problem by using the variable transformation method by Pal.

Integer programming problem, Integer programming problem is a common problem class where decision variables represent indivisibility or yes/no decisions. Integer Programming can also be interpreted asdiscrete optimizations which are extensively used in the areas of routing decisions, fleet assignment, and aircraft/aircrew scheduling. The cut mentioned is Gomory cut for integer programming problem with bounded variables [8].

Fuzzy goal programming approach is used for achieving highest degree of each of the membership goals to the extent possible by minimizing only the negative attainment variables. Euclidean distance function is then used to select the compromise optimal solution for both the DMs in the decision-making situation. A numerical example is provided in order to show the efficiency of the proposed approach.

In the literature, many researchers have focused to solve Bi-Level programming problems and fractional integer programming problems. In[1] A. Baky, Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach, then in[11] B. Pal, B. Moitra, and U. MaulikA, goal programming procedure for fuzzy multi objective linear fractional programming problem, [3] O. Emam and et al, On fuzzy bi-level multi-objective large scale integer quadratic programming problem,[4] O. Emam, Interactive approach to bi-level integer multi objective fractional programming problem,[10] M. Osman et al, Interactive Approach for Multi-Level Multi-Objective Fractional Programming Problems with Fuzzy Parameters,[6]. B. Ibrahim et al, Bi-level multi-objective programming problem with fuzzy demands: a fuzzy goal programming algorithm,[13] S.Pramanik and P. Pratim, Bi-level Linear Fractional Programming Problem based on Fuzzy Goal Programming Approach, [15] O.Saad and et al, On the solution of multi objective integer linear fractional programming problems with uncertain data,[17] M. Toksari and y. Bilim, Interactive fuzzy goal programming based on jacobian matrix to solve decentralized bi-level multi-objective fractional programming problems, [18] E. Youness and et al, Fuzzy bi-level multi-objective fractional integer programming,[16] V. Sharmaa and et al, A class of integer linear fractional bi level programming problems,[9] A. Nachammai and P. Thangaraj , Solving fuzzy linear fractional programming problem using metric distance ranking,[7] K. Lachhwani and A. Dwivedi, Bi-level and Multi-Level Programming Problems: Taxonomy of Literature Review and Research Issues,

This paper solves the bi-level integer linear fractional programming problems based on Fuzzy goal approach. the reminder of this paper is organized as follows: Section (2) formulated the bi-level integer linear fractional programming problem based on Fuzzy goal approach. Then, Section (3) proposed an algorithm for solving these programs. Section (4), illustrative example is given to demonstrate the proposed algorithm, Finally, Section (5) contains the conclusions.

To formulate the FGP Model of the BL-LFIP problem, the fuzzy goals of the objectives are determined by determining individual optimal solution. The fuzzy goals are then characterized by the associated membership functions which are transformed into fuzzy flexible membership goals by means of introducing over- and under deviational variables and assigning highest membership value (unity) as aspiration level to each of them. To elicit the membership functions of the decision vectors controlled by the FLDM, the optimal solution of the first-level LFIP problem is separately determined. A relaxation of the FLDM decisions is considered for avoiding decision deadlock.

The method of variable change on the under attainment- and over attainment variables of the membership goals associated with the fuzzy goals of the model is introduced to solve the problem efficiently by using linear goal programming (LGP) methodology.

2: Problem formulation and solution concept.

The bi-level integer fractional programming problem fuzzy goal (BLFIP) may be formulated as follows [4]:

Assume that there are two levels in a hierarchy structure with first-level decision maker (FLDM) and second-level decision maker (SLDM). Let the vector of decision variables $z = (x, y) \in R^n$ be partitioned between the two planners. The first-level decision maker has control over the vector $x \in R^{n1}$ and the second-level decision maker has control over the vector$y \in R^{n2}$, where $n = n1 + n2$. furthermore, assume that

$$F_i(x, y): R^{n1} \times R^{n2} \dashrightarrow R^{mi}, \quad i = 1, 2 \ ; \qquad (1)$$

Are the first-level and second-level vector objective functions, respectively. So the BL-LFP problem of minimization type may be formulated as follows:

[1st Level]

$$\underset{x}{\mathrm{Max}}\, F_1(x, y) = \max_x\big(f_1(x, y)\big), \qquad (2)$$

$$where\ x_2\ solves$$

[2nd Level]

$$\underset{y}{\mathrm{Max}}\, F_2(x, y) = \max_y\big(f_2(x, y)\big), \qquad (3)$$

Subject to

$$z\epsilon G = \{z = (x, y) \in R^n | A_1 x + A_2 y\ (\le)\ b, z \ge 0, b \in R^m, and\ integers\} \ne \emptyset,$$

where (4)

$$f_{ij}(x, y) = \frac{c_{ij} z + \alpha_{ij}}{d_{ij} z + \beta_{ij}}, \qquad (5)$$

where

$$i = 1\, for\ \ FLDM\ objective\ functions,$$
$$j = 2\, for\ SLDM\ objective\ functions,$$

and where

(i) $x = (x^F, y), \ y = (x, y^S)$,
(ii) G is the the bi-level convex constraints feasible choice set,
(iii) m_1 is the number of first-level objective functions,
(iv) m_2 is the number of second-level objective functions,
(v) m is the number of the constraints,
(vi) A_i: m × $n_i$matrix, i = 1, 2,
(vii) c_i, $d_i \in R^n$, $d_i x + \beta_i > 0$ for all z∈ G,
(viii) β_i, α_i are constants.

$$(FLDM)\ \max_x F_1(z) = \frac{c_1^t z + \alpha_1}{d_1^t z + \beta_1}, \qquad (6)$$

$$where x_2 solves$$

$$(SLDM)\ \max_y F_2(z) = \frac{c_2^t z + \alpha_2}{d_2^t z + \beta_2}, \qquad (7)$$

Subject to

$x \in [M],$ (8)

where $[M]$ is the convex hull of the feasible region M defined by (3) earlier. This convex hull is defined by [16]:

$$[M] = M_R^{(S)} = \{z \in R^n | A^{(s)} z \le b^{(s)}, z \ge 0\}, \quad (9)$$

and in addition,

$$A^{(s)} = \begin{bmatrix} A \\ . \\ h_1 \\ . \\ . \\ h_s \end{bmatrix}, \quad \text{and} \quad b^{(s)} = \begin{bmatrix} lb_1 \\ . \\ r_1 \\ . \\ . \\ r_s \end{bmatrix}, \quad (10)$$

$A^{(s)}$,$b^{(s)}$are the original constraint matrix A and the right-hand side vector b, respectively, with s-additional constraints.

3: Fuzzy Goal Programming Formulation of BL-LFIP.

In BL-LFIP problems, if an imprecise aspiration level is assigned to each of the objectives in each level of the BL-LFIP, then these fuzzy objectives are termed as fuzzy goals. They are characterized by their associated membership functions by defining the tolerance limits for achievement of their aspired levels.

3.1. Construction of Membership Functions

Since the FLDM and the SLDM both are interested of minimizing their own objective functions over the same feasible region defined by the system of constraints (2.4), the optimal solutions of both of them calculated in isolation can be taken as the aspiration levels of their associated fuzzy goals.

$Let\ (x^{ij}, y^{ij}; f_{ij}^{min}, j = 1, 2)\ and\ (x^{2i}, y^{2j}; f_{2j}^{min}, j = 1, 2)$be the optimal solutions of FLDM and SLDM objective functions, respectively, when calculated in isolation. Let $g_{ij} \ge f_{ij}^{min}$ be the aspiration level assigned to the +ijth objective $f_{ij}(x, y)$ (the subscript ij means that j = 1, 2 when i = 1 for FLDM problem, and j = 1, 2 when i = 2 for SLDM problem). Also, let $z^* = (x^F$, $y^s)$, $x^F = (x^*, y^*)$ and $y^S = (x^*, y^*)$, bethe optimal solution of the FLDM LFP problem. Then, the fuzzy goals of the decisionmakers objective functions at both levels and the vector of fuzzy goals of the decision variables controlled by first-level decision maker appear as

$$f_{ij}(x, y) \le g_{ij}, i = 1, 2, \quad j = 1, 2, \quad x = x^*, \quad (11)$$

Where≤ and "=" indicate the fuzziness of the aspiration levels and are to be understood as "essentially less than" and "essentially equal to".

Where (g_i),is the lower tolerance limit or lowest acceptable level of achievement for the membership function.

It may be noted that the solutions $(x^{ij}, y^{ij}), i = 1,2,\ j = 1,2\ , z^* = (x^*, y^*)$are usually different because the objectives of FLDM and the

Objectives of the SLDM are conflicting in nature. Therefore, it can reasonably be assumed as the upper tolerance limit u_{lm} of the fuzzy goal to the objective functions $f_{lm}(x, y)$. Then, membership functions $(f_i(x, y))$for the ith fuzzy goal can be formulated as:

$$\mu_{f_i}(f_i(x,y)) = \begin{cases} 1, & if\ (f_i(z)) \le g_i \\ \dfrac{u_i - f_i(x,y)}{u_i - g_i} & if\ \ g_i \le (f_i(z))f_i(z) \le u_i \\ 0, & if(f_i(z)) \le u_i \end{cases} \quad (12)$$

the optimal solution $z^* = (x^F, y^S)$ of the first-level LFP problem should be determined first. Following Pal et al. approach [11], the optimal solution

$z^* = (x^F, x^s)$ Could be obtained. It may be noted that any other approaches for solving LFP problems can be

used in solving the first-level ILFP problem. In Section 4, the FGP model of Pal et al. [11], for solving the first-level MOLFP problem, is presented to facilitate the achievement of $z^* = (x^F, y^S)$.

3.2. Fuzzy Goal Programming Approach.

In fuzzy programming approach, the highest degree of membership function is 1. So, as in [19], for the defined membership functions in (3.2) and (3.3), the flexible membership goals with the aspired level 1 can be presented as.

$$\mu_{f_i}(f_i(x,y)) + d_i^- - d_i^+ = 1, \quad i = 1,2,\dots,m_i, \quad (13)$$
$$\mu_{x_1^k}(x_1^k) + d_k^- - d_k^+ = 1, \quad k = 1,2,\dots,n_1,$$

or equivalently as(14)

$$\frac{u_i - f_i(x,y)}{u_i - g_i} + d_i^- - d_i^+ = 1, \quad i = 1,\dots,m_i,$$
$$\frac{x_1^k - (x_1^{k^*} - t_k^l)}{t_k^l} + d_k^{l-} - d_k^{l+} = 1, \quad k = 1,2,\dots,n_{1,}$$
$$\frac{(k_1^{k^*} + t_k^R) - x_1^k}{t_k^R} + d_k^{R-} - d_k^{R+} = 1, \quad k = 1,2,\dots,n_{1,}$$

where $d_k^- = (d_k^{L-}, d_k^{R-})$, $d_k^+ = (d_k^{L+}, d_k^{R+})$, and $d_i^-, d_k^{L-}, d_k^{R-}, d_i^+, d_k^{L+}, d_k^{R+} \ge 0$ with $d_i^- \times d_i^+ = 0, d_k^{R-} \times d_k^{R+} = 0$,and $d_k^{R-} \times d_k^{R+} = 0$ represent the underattainment- and over attainment , respectively, from the aspired levels.

In conventional GP, the under- and/or over attainment variables are included in the achievement function for minimizing them and that depend upon the type of the objective functions to be optimized. In this approach, the over-attainment variables for the fuzzy goals of objective functions, d+, and the over attainment -and the under attainment variables for the fuzzy goals of the decision variables,$d_k^{L-}, d_k^{L+}, d_k^{R-}$and d_k^{R+},are required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that any under-deviation from a fuzzy goal indicates the full achievement of the membership value [11].

It can be easily realized that the membership goals in (3.2) are inherently nonlinear in nature and this may create computational difficulties in the solution process. To avoid such problems, a linearization procedure is presented in the following section.

The FGP approach programming problems is extended here to formulate the FGP approach to bi-level linear fractional programming. Therefore, considering the goal achievement problem of the goals at the same priority level, the equivalent fuzzy bi level linear fractional goal programming model of the problem can be presented as:

$$\min z = \sum_{j=1}^{m_1} w_{1j}^{+} d_{1j}^{+} + \sum_{k=1}^{n_1} \left[w_k^L \left(d_k^{L+} + d_k^{L-} \right) + w_k^R (d_k^{R+} + d_k^{R-}) \right] \sum_{j=1}^{m_2} w_{2j}^{+} d_{2j}^{+} \qquad (15)$$

Subject to $\mu_{f1}\left(f_1(x,y)\right) + d_1^- - d_2^+ = 1,$

$$\mu_{f2}\left(f_2(x,y)\right) + d_2^- - d_2^+ = 1,$$
$$\mu_{x_1^k}\left(x_1^k\right) + d_k^- - d_k^+ = I, \quad k = 1,2,\dots,n_1,$$
$$A_1 x_1 + A_2 x_2 (\leq) b, \quad x \geq 0,$$
$$d_i^-, d_i^+ \geq 0, \ \ with\ d_i^- \times d_i^+ = 0,$$
$$d_k^-, d_k^+ \geq 0, \ \ with\ d_k^- \times d_k^+ = 0,$$

and the above problem can be rewritten as

$$\min \mathrm{z} = \sum_{j=1}^{m_1} w_{1j}^{+} d_{1j}^{+} + \sum_{k=1}^{n_1} [w_k^L (d_k^{L+}, d_k^{L-}) + w_k^R (d_k^{R+}, d_k^{R-})] + \sum_{j=1}^{m_2} w_{2j}^{+} d_{2j}^{+} \quad (16)$$

Subject to $\frac{u_1 - f_1(x,y)}{u_1 - g_1} + d_1^- - d_1^+ = 1,$

$\frac{u_2 - f_2(x,y)}{u_2 - g_2} + d_2^- - d_2^+ = 1,(17)$

$$\frac{x_1^k - \left(x_1^{k^*} - t_k^L\right)}{t_k^L} + d_k^{L-} - d_k^{L+} = 1, \quad k = 1,2,\dots,n_1,$$
$$\frac{\left(x_1^{k^*} + t_k^R\right) - x_1^k}{t_k^R} + d_k^{R-} - d_k^{R+} = 1, \quad k = 1,2,\dots,n_1,$$
$$A_1 x + A_2 y\ (\leq)\ b\,, \quad z \geq 0,$$
$$d_i^-, d_i^+ \geq 0\ with\ d_i^- \times d_i^+ = 0, \quad i = 1,2\dots,m_i,$$
$$d_k^{L-}, d_K^{L+}\ \geq 0, with\ d_k^{L-} \times d_k^{L+} = 0\,, k = 1,2,\dots,n_1,$$
$$d_k^{R-}, d_k^{R+} \geq 0, with\ d_k^{R-} \times d_k^{R+} = 0, \quad k = 1,2,\dots,n_1.$$

3.3. Linearization of Membership Goals

Following Pal et al. [11], the I jth membership goal in3.5can be presented as

$$L_i u_i - L_i f_i(x,y) + d_i^- - d_i^+ = 1 \quad where\ \ L_i = \frac{1}{u_i - g_i}.$$

Introducing the expression of$f_i(x,y)$ from (2.5),the above goal can be presented as:

$$L_i u_i - L_i \frac{c_i z + \alpha_i}{d_i z + \beta_i} + d_i^- - d_i^+ = 1, \quad where\ \ L_i = \frac{1}{u_i - g_i}. \qquad (18)$$

$\Rightarrow L_i u_i (d_i z + \beta_i) - L_i (c_i z + \alpha_i) + d_i^- (d_i z + \beta_i) - d_i^+ (d_i z + \beta_i) = (d_i + \beta_i)(19)$

$\Rightarrow -L_i (c_i z + \alpha_i) + d_i^- (d_i + \beta_i) - d_i^+ (d_i z + \beta_i) = [1 - L_i u_i](d_i z + \beta_i)$

$\Rightarrow -L_i (c_i z + \alpha_i) + d_i^- (d_i z + \beta_i) - d_i^+ (d_i + \beta_i) = L_i^{/} (d_i x + \beta_i),\ where\ \ L_i^{/} = 1 - L_i u_i$

$\Rightarrow \left(-L_I c_i - L_i^{/} d_i\right) z + d_i^- (d_i z + \beta_i) - d_i^+ (d_i z + \beta_i) = L_i \alpha_i + L_i^{/} \beta_i$

$\Rightarrow C_i z + d_i^- (d_i z + \beta_i) - d_i^+ (d_i z + \beta_i) = G_{i,}$

Where

$$C_i = -L_i c_i - L_i^{/} d_i, \qquad G_i = L_i \alpha_i + L_i^{/} \beta_i. \qquad (20)$$

Now, using the method of variable change as presented by Kornbluth and Steuer, Pal et al. (11), the goal expression in 3.9can be linearized as follows.

Let $D_i^- = d_i^-(d_i z + \beta_i), and\ D_i^+ = d_i^+(d_i z + \beta_i)$;the linear form of the expression in 3.9is obtained as:

$$C_i z + D_i^- - D_i^+ = G_i (21)$$

With $D_i^-, D_i^+ \geq 0 and\ D_i^- \times D_i^+ = 0$ since$d_i^-, d_i^+ \geq 0\ and\ d_i z + \beta_i > 0$.

Now, in making decision, minimization of d_i^+ means minimization of $D_i^+ = d_i^+(d_i z + \beta_i)$,which is also a non-linear one.It may be noted that when a membership goal is fully achieved, $d_i^+ = 0$ and when its achievement is zero, $d_i^+ = 1$are found in the solution 26. So, involvement of $d_i^+ \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\frac{D_i^+}{d_i z + \beta_i} \leq 1, \text{ that is,} -d_i z + D_i^+ \leq \beta_i. \quad (22)$$

Here, on the basis of the previous discussion, it may be pointed out that any such constraint corresponding to d_i^-does not arise in the model formulation.

Therefore, under the framework of minimize GP, the equivalent proposed FGP model of problem (3.7) becomes

$$\min z = \sum_{j=1}^{m_1} \omega_1^+ D_1^+ + \sum_{k=1}^{n_1}[\omega_k^L(d_k^{L+} + d_k^{L-}) + \omega_k^R(d_k^{R+} + d_k^{R-})] + \sum_{j=1}^{m_2} \omega_2^{+;} D_2^+ (23)$$

$$\text{subject to } C_i\text{x}+D_1^- - D_1^+ = G_1,$$

$$C_2 x + D_2^- - D_2^+ = G_{2,}$$

$$\frac{x_1^k - (x_1^{k^*} - t_k^L)}{t_k^L} + d_k^{L-} - d_k^{L+} = 1, \quad k = 1,2, \dots, n_1,$$

$$\frac{(x_1^{k^*} + t_k^R) - x_1^k}{t_k^R} + d_k^{R-} - d_k^{R+} = 1, \quad k = 1,2, \dots, n_1,$$

$$-d_i + D_i^+ \leq \beta_i, \quad i = 1,2, \dots, m_i,$$

$$A_1 x + A_2 y\ (\leq) b, \quad z \geq 0,$$

$$D_i^-, D_i^+ \geq 0, \quad i = 1,2, \dots, m_i,$$

$$d_k^{L-}, d_k^{L+} \geq 0 \quad \text{with } d_k^{L-} \times d_k^{L+} = 0, \quad k = 1,2, \dots, n_1,$$

$$d_k^{R-}, d_k^{R+} \geq 0, \quad \text{with} d_k^{R-} \times d_k^{R+} = 0, \quad k = 1,2, \dots, n_1,$$

where Z represents the fuzzy achievement function consisting of the weighted over-attainment variables $D_i^+$of the fuzzy goals g_i and the underattainment and the over-attainment variables $d_k^{L-}, d_k^{R-}, d_k^{L+}$, and d_k^{R+},$k = 1, 2, \dots, n_1$for the fuzzy goals of the decision variables $x^1, x^2, x^3, \dots, x^n$,

where the numerical weights ω_i^+, ω_k^L, and ω_k^R relative importance of achieving the aspired levels of the respective fuzzy goals subject to the constraints set in the decision situation.

To assess the relative importance of the fuzzy goals properly, the weighting scheme can be used to assign the values to ω_i^+ andω_i^-. In the presentformulation, the values of ω_i^- andω_1^- are determined as:

$$\omega_i^+ = \frac{1}{u_i - g_i}, \quad i = 1,2, \dots, m_i, \quad (24)$$

$$\omega_k^L = \frac{1}{t_k^L}, \quad \omega_K^R = \frac{1}{t_k^R}, \quad k = 1,2, \dots, n_1,$$

The FGP model (3.13) provides the most satisfactory decision for both the FLDM and the SLDM by achieving the aspired levels of the membership goals to the extent possible in the decision environment. The solution procedure is straightforward and illustrated via the following example.

4. The FGP Model for MOLFP Problems

In this section, the FGP model of Pal et al. [11], for solving the first-level MOLFP problem, is presented here to facilitate the achievemento $f\ z^* = (x^F, y^S)$. This solution is used to elicit theMember ship functions of the decision vectors$x = (x^1, x^2, x^3, \ldots, x^{n_1})$, that included in the FGP approach for solving BLLFP problems proposed in this article.

The first-level MOLFP problem is

Min $\quad F_1(x,y) = \text{Min}\big(f_1(x,y)\big)$(23)

subject to Z∈G $=\{z = (x,y) \in R^n | A_1x + A_2y(\leq)\, b, z \geq 0, b \in R^m\} \neq \emptyset.$ (24)

And the FGP model of Pal et al. is

$$\min \quad Z=\sum_{j=1}^{m_1} w_{1j}^{+} D_{1j}^{+} \tag{25}$$

subject to $\quad C_1x + D_1^- - D_1^+ = G$,(26)

$$-d_1z + D_1^+ \leq \beta_1,$$

$$A_1x + A_2y \leq b, \qquad z \geq 0,$$

$$D_1^-, D_1^+ \geq 0,$$

5. The FGP Algorithm for BL-MOLFP Problems

Following the above discussion, we can now construct the proposed FGP algorithm for solving the BL-ILFP problems.

Step (1): Convert the problem BLIFP into the equivalent problem BLFIP, go to Step 2.

Step (2): Calculate the individual minimum and maximum of each objective function in the two levels underusing the Gomory cut for the constraints.

Step (3): Set the goals and the upper tolerance limits for all the objective functions in the two levels.

Step (4): Elicit the membership functions $\mu_{f1}\big(f_1(x,y)\big)$in the first level.

Step (5): Formulate the Model (4) for the first level LFIP problem.

Step (6): Solve the Model (4) to get $z^* = (x^*, y^*)$.

Step (7): Set the maximum negative and positive tolerance values on the decision vector (x).

Step (8): Elicit the membership functionsμ_{x^k} for decision vector.

Step (9): Elicit the membership functions $\mu_{f2}\big(f_2(x,y)\big)$ in the second level.

Step (10): Formulate the Model (23) for the BL-LFIP problem.

Step (11): Solve the Model (23) to get the satisfactory solution$z^* = (x^F, y^S)$ of the BL-LFIP problem.

6. Numerical Example

To demonstrate proposed FGP procedure, consider the following bi-level integer linear fractional programming problem:

[1st Level]

$$\min_{x} \quad \frac{2x + 3y}{x + 4y + 6}$$
where y solves
$$\min_{y} \frac{3x + 4y}{6x + 4y + 3}$$
subject to
$$\mathrm{x + y \le 5;}$$
$$\mathrm{x + y \le 10,}$$
$$\mathrm{x + y \le 7,}$$
$$\mathrm{x \le 3,}$$
$$\mathrm{x,y \ge 0 \text{ and integers;}}$$

	f_1	f_2
c	(2,3)	(3,4)
d	(1,4)	(6,4)
α	0	0
β	6	3
Max	0.69	0.87
Min	0	0
u	0.69	0.87
g	0	0
L	1.449	1.149
$L^{/}$	1	1
G	6	3
C	(1.898 ,0.347)	(−2.553 ,0.596)
W	1.449	1.149

Table1: Coefficientsα_i, β_i , $c_i$and $d_i$second-level objectives of the BL-ILFP problem.
The FGP model for the first-level linear fractional programming problem is obtained as

$$\min \ z = d_{1n} ;$$
$$\text{subject to} 1.889\text{x}+0.332\text{y}+d_{1n} - d_{1p}\text{=6;}$$
$$-x - 4d_{1n} \ \le 6;$$
$$x + y \le 5;$$
$$3x + y \le 10;$$
$$2x + y \le 7;$$
$$x \le 3;$$

The solution of the first-level will be obtained as:
Z=0.001,$d_{1n} = 0.001$, $d_{1p} = 0$, $(x = 3 \, , y = 1)$.
the FGP model for the second-level linear fractional programming problem is obtained as:

$$\min \ z = d_{1n} + d_{2n} ;$$
$$1.889x + 0.332y + d_{1n} - d_{1p} = 6;$$

$$-2.55x + 0.6y + d_{2n} - d_{2p} = 3;$$
$$-x - 4d_{1n} \leq 6;$$
$$-6x - 4y + d_{2n} \leq 3;$$
$$x + y \leq 5;$$
$$3x + y \leq 10;$$
$$2x + y \leq 7;$$
$$x = 3;$$

The solution of the second-level will be obtained as:

Z=10.05 $d_{1n} = 0.001$, $d_{2n} = 10.05$, $d_{1p} = 0$, $d_{2p} = 0$,

$$(x = 3, \quad y = 1).$$

7. Conclusion.

This paper proposed an algorithm for solving the bi-level integer linear fractional programming problem by a fuzzy goal approach. The solution algorithm described by two main phases: first the solution algorithm should avoid the complexity of non-convexity nature of the constraint set by constructing the convex hull equivalent to the original set of constraints using the cutting-plane algorithm, and then the solution process introduces the pal transformation method to obtain linearize the member ship goal for the integer solution. At the second phase, from this obtained integer solution, the fuzzy goal programming approach is used to solve the problem by minimizing only negative deviational variables. Then, the Euclidean distance function is used to identify the optimal solution. We can apply the concept to decentralized bi-level multi-objective and multi-level multi-objective fractional programming problems based on real-life decision-making problems.

8. References.

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