Novel Method for Optimizing Traffic Police Manpower by Standby and Patrol

Mingyue Miao[1], Said Easa[2], Jianying Chen[3], Jun Zhang[4], Qiang Li[5], Yanqun Yang[3*]

[1] *School of Urban Economics and Public Administration, Capital University of Economics and Business, Beijing, 100070, China.*

[2] *Department of Civil Engineering, Ryerson University, Toronto, Ontario M5B 2K3, Canada.*

[3] *College of Civil Engineering, Fuzhou University, Fuzhou, Fujian, 350108, China.*

[4] *School of Information, Capital University of Economics and Business, Beijing, 100070, China.*

[5] *Traffic Police Detachment, Capital International Airport, Beijing, 100621, China.*

**Correspondence should be addressed to Yanqun Yang; yangyanqun@fzu.edu.cn*

Abstract：*The purpose of this research paper is to find out a quantitative method that could assist traffic police department with determining minimum patrol and standby staffing and optimizing shift schedule to meet current performance benchmarks. The average events rate reflects the overall traffic situation over a certain period of time. By Fault Tree Analysis (FTA) method, average events occurrence rate could be expressed by a linear equation, which combines the number of traffic accidents, congestion events, and serious violation events with their key importance coefficients. Next, the number of patrol team and the sum of the minimum number of patrol and standby police could be obtained by applying Queueing model and Poisson Distribution model, based on comparing and analyzing both models by average events occurrence rate. These variables mentioned will be used to establish constraint equations in Integer Programming (IP) model. Finally, it is provided to the traffic police detachments or stations, especially those consisted of patrol police and clerical police, to a relatively simple and quantitative way to optimize police resources based on traffic conditions of their precinct.*

Keywords*: Traffic police; Police resources; Queueing model; Poisson Distribution model*

Introduction

Traffic police are police officers who enforce road rules, prevent driving violations, and handle traffic accidents (Adler et al., 2014). They play an important role in maintaining efficient and safe operation of road networks. In recent years, the number of motor vehicles has been increasing rapidly in China. However, the number of traffic police has almost been the same or increasing with an unmatched low growth rate, mainly given to the economic reason. Thus, the police resources are limited. However, there are many emergency situations in road traffic, which leads to increasing work pressure on traffic police. They usually have to work for a long time and shift from here to there, which brings serious impact on their health and performance (Hickman et al., 2011; Vila, 2006; Violanti et al., 2012). Fatigue caused by police patrols due to departmental policies and practices may reduce an individual's ability to reduce organizational performance (Vila et al., 2002). The allocation of limited police patrol resources is a difficult problem for police commanders, so a reasonable traffic police shifting schedule is particularly important.

Without considering the addition of electronic equipment and other facilities, previous studies on police optimization were mainly focused on two aspects. One was how to optimize the deployment of the police force, so that it was in line with the distribution of crimes and emergencies. The other one was how the police arrangement shifts. There are different views on the scheduling mode that suits police officers the best, and it is safe to say that the generally accepted opinion has not been established yet. Researches on personnel and vehicle scheduling methods mainly include mathematical programming, such as integer programming, linear programming, dynamic programming, goal programming, and others include simulation, constraint programming and queuing (Van den Bergh et al., 2013).

Studies on police patrol can be traced back to the early 1970s. The researches mainly used queuing model to solve the problem of police patrol scheduling (Keskin et al., 2012). Regarding the practical application of police deployment, some mature models have been formed. For example, the hypercube queuing model has been widely used in police patrol area. It was a random queuing model that explains the distribution of patrol cars to areas through probabilistic models (Wu & Lou, 2014). Another well-established model was the Patrol Car Allocation Model (PCAM), developed by Chaiken & Dormont (1978). PCAM was designed to help police managers specify the number of patrol cars that should be on duty at different times of each day of the week in each region, but the model was only applicable to a patrol area that has been shaped and sized. As a result, the workload of the police was seriously uneven in practical applications. On the basis of PCAM, D'Amico et al. (2002) used the simulated annealing method to better solve the problem of unbalanced workload between polices. However, PCAM can only solve the scheduling problem of a single car. Thus, Green developed a multi-priority queuing model to accommodate multiple patrol car dispatches (Green, 1984; Green & Kolesar, 1984; Green & Kolesar, 1989). In addition, Ozbay et al. (2013) used integer programming model to solve vehicle allocation problem considering resource demand and probability of event occurrence. Yin (2008) dealt with the randomness of events based on scenarios, and proposed a mixed integer nonlinear programming model, with the goal of finding a good truck distribution plan. Lou et al. (2011) proposed a mixed integer nonlinear model for optimal vehicle allocation. Mixed integer linear programming models were proposed by several researchers to solve the personnel assignment problem (Bassett, 2000; Wu & Sun, 2006). In addition, a goal programming was used to optimize personnel scheduling (Basu & Ghosh, 1997; Mustapar et al., 2017; Sharma et al., 2007; Todovic et al., 2015). Pal et al. proposed a fuzzy goal programming (FGP) model to solve the problem of deployment of urban patrol manpower (Pal et al., 2009; Pal et al., 2012). Kolesar et al. (1975) used a queuing model and an integer linear programming model to arrange patrol cars to meet specific service standards per hour. Alfares (2007) used queuing model to estimate personnel demand per hour, and then constructed a integer programming model to find the best employee scheduling scheme to meet labor demand at minimum cost.

At present, the commonly used solving methods for scheduling problems are genetic algorithm (GA), simulated annealing algorithm (SA), ant colony algorithm (ACA), particle swarm optimization (PSO) and artificial neural network (ANN) (Yanan & Huayu, 2012). Many researchers used simulated annealing algorithms to find the best or near optimal police patrol scheduling solution (D'Amico et al., 2002; Zhang & Brown, 2014). Genetic algorithms was used to solve the deployment of police manpower in specific areas or situations, but these study did not address specific traffic police and traffic conditions (Kou & Liu, 1996; Nag, 2014). Yanan & Huayu (2012) established a mathematical model of the patrolman scheduling problem, and proposed a genetic simulated annealing hybrid algorithm to solve the patrol scheduling problem.

Generally speaking, previous studies on the model for handling patrol car deployment were almost entirely based on queuing model or integer programming or a combination of the two, and focused on the police of a specific type of job, such as police patrol, without considering the standby police and their links to the patrol police. With limited police resources, it was often necessary to dispatch police resources located on various service platforms to certain important locations for law enforcement. In some countries and regions, office police also have law enforcement powers as road patrol police, and the later sometimes take part in patrols as complementary or standby forces, such as China, South Korea, etc. Due to the division of responsibilities, the police have different orders in different jobs in response to emergencies. Therefore, the queuing programming method was improved according to the different tasks and sequences of patrol police and standby police in dealing with various traffic events, and a new method to optimize the manpower of traffic police was proposed to optimize the dispatching of patrol personnel.

Methodology

Traffic events are mainly categorized to traffic accidents, serious traffic violation events and traffic congestion, some of which may cause traffic policemen or teams to be dispatched to deal with and eliminate the impact of events on road network. Upon inputting average waiting time (w_q), average service rate (u) and average events occurrence rate (λ), Queueing model can be used to calculate the corresponding minimum number of traffic police (Figure 1). The average events occurrence rate is determined by a variety of events that may result in dispatched traffic police. Fault Tree Analysis (FAT) can help to establish an equation expressing the relationship between the value of λ and the number of traffic events after reprocessing data to ensure the independence between the data entering into FAT. These traffic events are considered to be following the Poisson distribution. When the cumulative probability reaches 0.95, the number of police officers required can be considered as the minimum demand, including the patrol police and the standby police. Figure 1 shows the variables needed, calculation process and outputs.

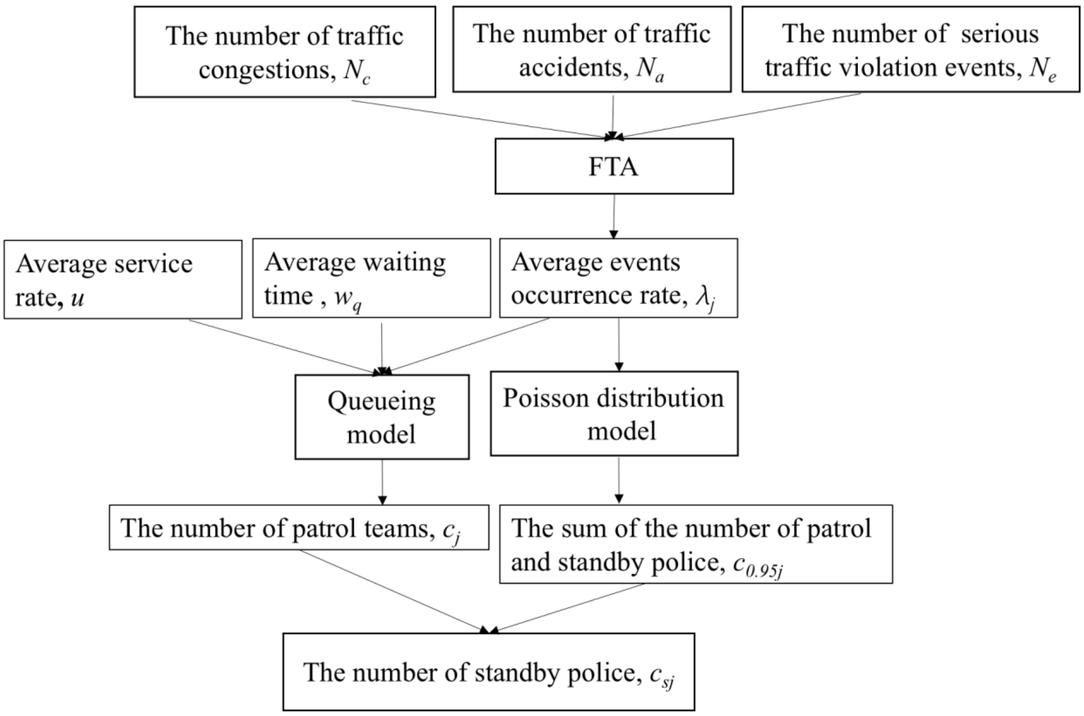
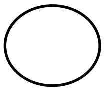


Figure.1 Logic of models

How to get the value of λ: Fault Tree Analysis. Fault Tree Analysis (FTA) is often used to quantitatively analyze logic relationships between independently basic events. In fault tree diagram, logic gate (or named operator) as a basic symbol (Shown as Table 1) to depict and interrelate the relationships among events. Each gate has inputs and an output; the gate inputs are the lower events and the output is a higher fault event or the top event. In addition to computing reliability measures of a system, it is often useful to determine which parts of a system contributes to the measure the most.

Table.1 Symbol in fault tree diagram

Symbol	Explanation
	If the occurrence of either input event causes the output event to occur, then these input events are connected using an OR gate.
	If both input events must occur in order for the output event to occur, then they are connected by an AND gate.
	Basic event, a basic initiating event.
	Top or intermediate event

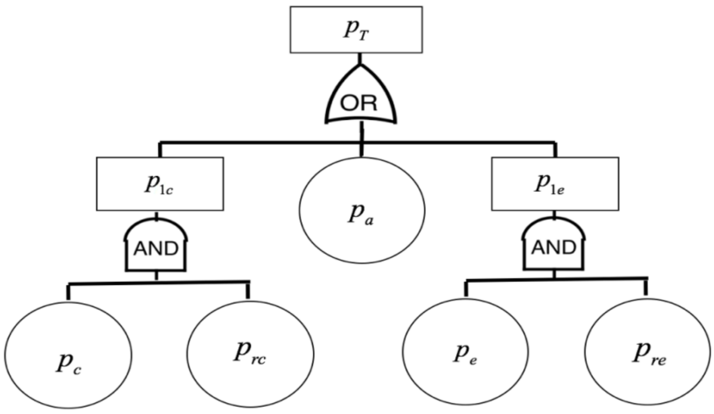


Figure.2 Fault tree diagram

In the initial data processing process, traffic congestion events related to traffic accidents were removed to ensure that various input events are independent of all others. Fault tree diagram (Shown as Figure 2) may be converted to be Equation (1) as follows:

$$p_T = p_a + p_{1c} + p_{1e} = p_a + p_c p_{rc} + p_e p_{re} \quad (1)$$

$$g_a = \frac{\partial p_T}{\partial p_a} = 1 \quad (2)$$

$$g_c = \frac{\partial p_T}{\partial p_c} = p_{rc} \quad (3)$$

$$g_e = \frac{\partial p_T}{\partial p_e} = p_{re} \quad (4)$$

Where P_T denotes the occurrence probability of top event: traffic police dispatched. P_{1c}, P_{1e} are probabilities of intermediate events occurrence. P_a, P_c, P_e are occurrence probabilities of traffic accidents, traffic congestion events and traffic violation events occurrence, respectively. P_{rc}, P_{re} are occurrence probabilities of dispatch of traffic police caused by traffic congestion and traffic violation, respectively. g_a, g_c, g_e denotes the key importance of traffic accidents, traffic congestion and traffic violation corresponding to dispatch of traffic police, respectively.

In terms of average events occurrence rate λ, based on the above Fault Tree Analysis, an equation is as follows:

$$\lambda = g_a N_a + g_c N_c + g_e N_e = N_a + p_{rc} N_c + p_{re} N_e \quad (5)$$

How to get c_j: Queueing model. According to the Queueing model, the average waiting time of the incident parties before traffic policemen arriving is:

$$w_q = \frac{L_q}{\lambda} = \frac{(c\rho)^c \rho}{c!(1-\rho)^2 \lambda} P_0 = \frac{(c\rho)^c \rho}{c!(1-\rho)^2 \lambda} [\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)}]^{-1} \quad (6)$$

Where w_q denotes average waiting time, L_q denotes Length of queueing, λ denotes average events occurrence rate, c denotes the number of teams regularly composed of two traffic policemen, ρ denotes intensity of incidents being dealt with. P_0 denotes the probability that there is no event being handled:

$$P_0 = [\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)}]^{-1} \quad (7)$$

$$\rho_j = \frac{\lambda_j}{c_j u} \quad (8)$$

Where ρ_j denotes intensity of events being dealt with within hour j. λ_j denotes average number of events occur per hour j. u denotes the average service rate of a single service (a single police patrol team). c_j denotes the number of teams regularly composed of two traffic policemen per hour j.

The traffic management department promised in public that the average waiting time for incidental parties normally should not exceed 15 minutes, that means $w_q \le 0.25$, and formula (6) can be changed to be formula (9).

$$\frac{(c\rho)^c \rho}{c!(1-\rho)^2 \lambda} [\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!(1-\rho)}]^{-1} \le 0.25 \quad (9)$$

If substituting formula (7) into formula (8), formula (9) can be easily gotten as follows:

$$\frac{(\frac{\lambda_j}{u})^{c_j}\frac{1}{c_j u}}{c_j!(1-\frac{\lambda_j}{c_j u})^2}[\sum_{k=0}^{c_j-1}\frac{(\frac{\lambda_j}{u})^k}{k!}+\frac{(\frac{\lambda_j}{u})^{c_j}}{c_j!(1-\frac{\lambda_j}{c_j u})}]^{-1}\le 0.25 \qquad (10)$$

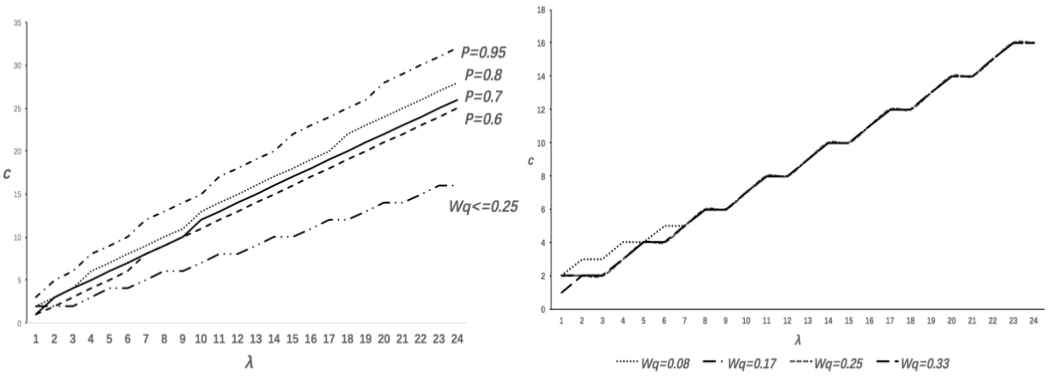
Since the inequality (10) is not easy to solve analytically, the following steps are recommended:

Step 1: Let $c_j > \frac{\lambda_j}{u}$ and be the smallest integer. Steady-state considerations can be achieved for the queueing process.

Step 2: Calculate w_q according to inequality (10). If $w_q \le 0.25$, terminate the algorithm. At this time, c is the minimum required number of incident handling teams, and record the corresponding waiting time w_q; if $\mathrm{w_q} > 0.25$, goes to Step 3.

Step 3: Let c_j=c_j+1, go back to step 2.

How to get c_{sj}: Poisson distribution. The Poisson distribution is suitable for describing the number of random events occurring within a unit of time. *P* denotes the cumulative probability of occurrence of *k* traffic events. For each hour, there are different amount of events occurring. When *P* is equal to 0.6, 0.7, 0.8, 0.95, respectively, values of *c* corresponding to the maximum, *k* can be obtained as shown Figure 3a.



(a) (b)

Figure.3 Values of c under various λ (a) and Wq (b)

Furthermore, from Figure 3a, it also shows that there are some cross points existing between lines drawn by following Queueing model and the Poisson distribution when the value of λ is small. After that, as λ increases, the demand of police calculated by Poisson distribution is significantly higher than that calculated by Queueing model. However, in any case, the line corresponding to 'w_q<=0.25' is always below the '*P*=0.95's.

The time waiting for the arrival of traffic police is generally expected to be limited within 20 minutes in cities of China. Sometimes, depending on features of events and occurring in different areas, 5 minutes, 10 minutes or 15 minutes, or even 20 minutes may be required. Obviously, the shorter the waiting time, the more traffic police are needed to deploy. As can be seen from Figure 3b, when the value of λ is small (less than 8), the gap between lines is distinguishable. As λ increases, the gap becomes smaller and smaller. Reasonably, when there are more incidents, the difference in demand of police manpower caused by the short time gap is insignificant.

Based on the analysis above, the total number of traffic police at working, including patrol police on road and standby police at office should be at least adequate to deal with no more than 95% of events. Therefore, $c_{0.95j}$ is used to denote the minimum in demand of traffic police working in a certain period of each hour. To determine the number of patrol police on roads and the number of standby police officers, formula (10) and (11) will be used respectively.

$$c_{sj} = c_{0.95j} - c_j \tag{11}$$

Where $c_{0.95j}$ is corresponding to 0.95 of the cumulative probability per hour j; c_{sj} denotes the minimum number of standby police at office per hour j.

Each patrol should be in the right place on the road so that it will take the least amount of time to reach the location where the event occurs. Information service, data processing and other jobs are done by the standby police force, and they also have patrol missions. Of course, according to the current mechanism, if the incident happens to be surprisingly large, commander center can immediately call the nearby traffic police to come back to work.

Application

Beijing Capital International Airport (BCIA) is currently ranked as the second largest airport in the world. Wikipedia cited 2016 data, that the passenger throughput of Capital Airport exceeded 94.39 million passengers, second only to the United States' Atlanta International Airport. It is sure that the traffic police there have undertaken a heavy task everyday. Normally, once an emergency on road takes place within the territory of BCIA (Figure 4), police patrolmen on road respond at first and must arrive at the site within 15-minute since the order received. If the number of policemen is not enough, the standby police at office will be informed as supplementary police force.

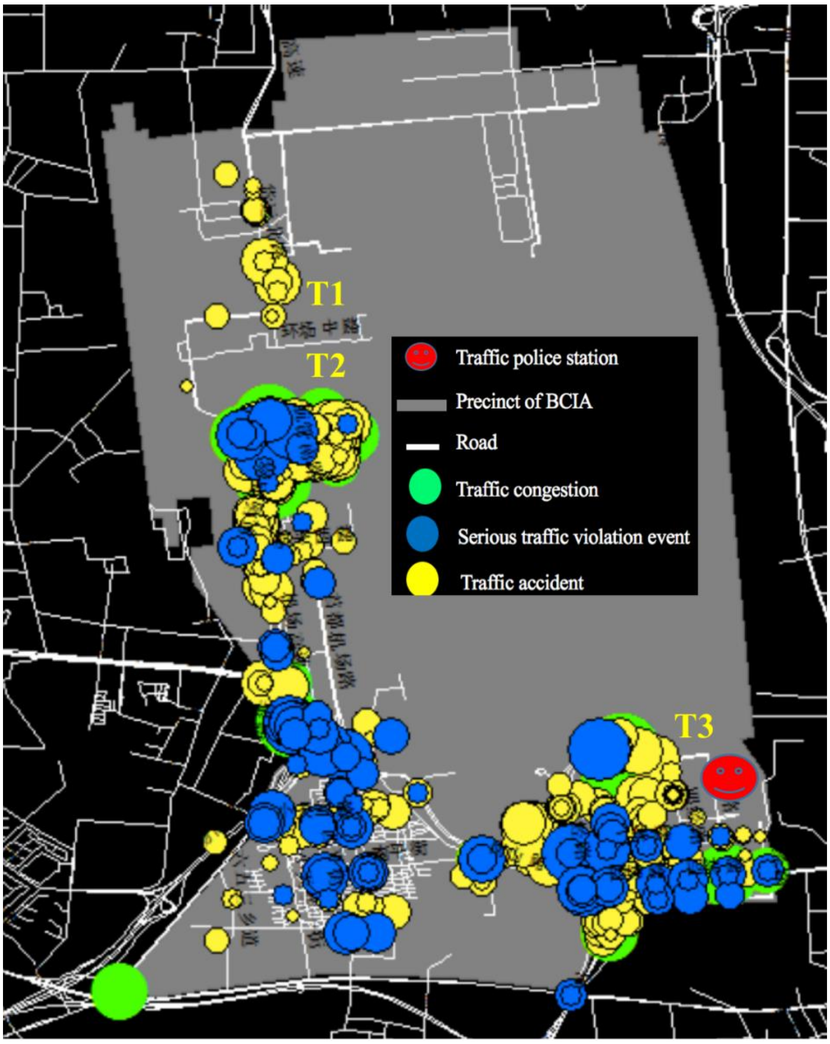


Figure.4 Map of traffic events at BCIA

It is one of goals that the sum of the number of patrol and standby police should be enough to handle almost all events for corresponding interval. The other one is that traffic police arrives at scenes as soon as possible.

Calculating the number of patrol and standby police. Through formula (10), formula (11), values of c_j and c_{sj} were obtained as Table 2 and Table 3 as following:

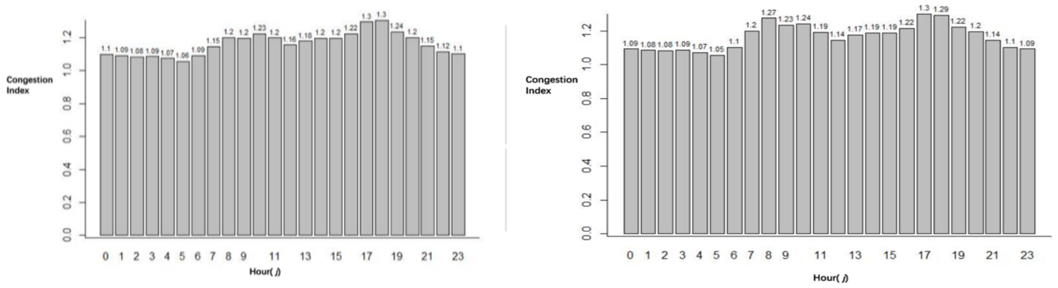
Table.2 Values of c_j and c_{sj} on weekday

j	λ_j	c_j	$c_{0.95j}$	c_{sj}
0:00-1:00	1.26	2	3	1
1:00-2:00	1.32	2	3	1
2:00-3:00	1.25	2	3	1
3:00-4:00	1	2	3	1
4:00-5:00	1.23	2	3	1
5:00-6:00	1.15	2	4	2
6:00-7:00	1.47	2	6	4
7:00-8:00	1.69	2	8	6
8:00-9:00	1.55	2	6	4
9:00-10:00	1.38	2	4	2
10:00-11:00	1.54	2	4	2
11:00-12:00	1.39	2	5	3
12:00-13:00	1.49	2	5	3
13:00-14:00	1.46	2	4	2
14:00-15:00	1.4	2	8	6
15:00-16:00	1.34	2	5	3
16:00-17:00	1.4	2	7	5
17:00-18:00	1.62	2	5	3
18:00-19:00	1.56	2	5	3
19:00-20:00	1.38	2	4	2
20:00-21:00	1.33	2	4	2
21:00-22:00	1.44	2	4	2
22:00-23:00	1.61	2	4	2
23:00-24:00	1.4	2	4	2

Table. 3 Values of c_j and c_{sj} on weekend

j	λ_j	c_j	$c_{0.95j}$	c_{sj}
0:00-1:00	1.14	4	6	2
1:00-2:00	1	4	6	2
2:00-3:00	1.4	4	8	4
3:00-4:00	1	4	6	2
4:00-5:00	1	4	6	2
5:00-6:00	1.3	4	6	2
6:00-7:00	1.2	4	6	2
7:00-8:00	1.27	4	6	2
8:00-9:00	1.44	4	8	4
9:00-10:00	1.13	4	6	2
10:00-11:00	1.56	4	8	4
11:00-12:00	1.59	4	6	2
12:00-13:00	1.38	4	8	4
13:00-14:00	1.27	4	6	2
14:00-15:00	1.11	4	6	2
15:00-16:00	1.48	4	6	2
16:00-17:00	1.31	4	8	4
17:00-18:00	1.18	4	6	2
18:00-19:00	1.25	4	6	2
19:00-20:00	1.23	4	6	2
20:00-21:00	1.38	4	6	2
21:00-22:00	1.36	4	8	4
22:00-23:00	1.42	4	8	4
23:00-24:00	1.14	4	8	4

Amap, who owns the most widely used navigation product in China, often uses Congestion Index (CI) to measure the traffic congestion. The CI is equal to v_{fj} / v_j, where v_{fj} is the highest speed of vehicles during hour j, v_j is the average speed of vehicles during hour j. The CI values in BCIA by hours on weekday (Figure 5a) and weekend (Figure 5b) are as follows:



(a) (b)

Figure.5 CI varies in BCIA by hours on weekday (a) and weekend (b)

As can be seen from the Figure 5, traffic congestion on road network in BCIA is evident at traffic peaks in the morning and evening. Considering actual traffic situation, when calculating the number of standby police, traffic police is required to take less than five-minute to arrive at occurring place from 7:00 am to 7:00 pm every day, that means $w_q \leq 0.08$. For the rest of time periods, it takes less than fifteen- minute, that means $w_q \leq 0.25$. Table 4 shows those key parameters' values needed by establishing IP model.

Table.4 Key parameters' values

Definition	Minimum amount in the number of patrol or standby police (2c)
Number of police patrolmen on road on weekday for day shift	4
Number of police patrolmen on road on weekday for night shift	4
Number of standby police on road on weekday for day shift	10
Number of standby police on road on weekday for night shift	6
Number of police patrolmen on road on weekday for day shift	4
Number of police patrolmen on road on weekend for day shift	4
Number of police patrolmen on road on weekend for night shift	4
Number of standby police on road on weekend for day shift	6
Number of standby police on road on weekend for night shift	4

Scheduling and optimizing formulation. Taking into account some actual conditions and habits of the traffic police at work, the main reference factors for IP model's constraints are as follows:

Factor 1: The police's working time can be divided into two parts: day shift time is 8:30-17:30, and the night shift time is 17:30-8:30.

Factor 2: Every police patrol team gets started to work only for day shift time on the first day, and works for 24 hours (a day shift and a night shift) on the second day. Then, they can get two consecutive days off.

Factor 3: The standby police and patrol traffic police is relatively independent.

Factor 4: All of standby policemen regularly have two days off every one week.

Factor 5: All of standby policemen take on one night-shift per week.

From Table 4, whether business day or weekend, whether day shift or night shift, the minimum number of patrol police is equal to four. Therefore, in combination with the Factor 2, at least sixteen police patrol officers are needed. Their weekly schedule is shown in Table 5.

Table.5 Schedule of the patrol police

Day of week	Patrol team for day shift	Patrol team for night shift
Monday	Team 1 Team2	Team 7 Team 8
Tuesday	Team 3 Team 4	Team 1 Team 2
Wednesday	Team 5 Team 6	Team 3 Team 4
Thursday	Team 7 Team 8	Team 5 Team 6
Friday	Team 1 Team 2	Team 7 Team 8
Saturday	Team 3 Team 4	Team 1 Team 2
Sunday	Team 5 Team 6	Team 3 Team 4

With reference to Factor 1, 3, 4, and 5, on the basis of Table 2 and Table 3, IP model can be written as follows:

$$\min G = \sum_{n=1}^{7} x_{dn} \qquad (12)$$

s.t

$$x_{d1} + x_{d4} + x_{d5} + x_{d6} + x_{d7} \geq 10$$

$$x_{d1} + x_{d2} + x_{d5} + x_{d6} + x_{d7} \geq 10$$

$$x_{d1} + x_{d2} + x_{d3} + x_{d6} + x_{d7} \geq 10$$

$$x_{d1} + x_{d2} + x_{d3} + x_{d4} + x_{d7} \geq 10$$

$$x_{d1} + x_{d2} + x_{d3} + x_{d4} + x_{d5} \geq 10$$

$$x_{d6} + x_{d2} + x_{d3} + x_{d4} + x_{d5} \geq 6$$

$$x_{d7} + x_{d2} + x_{d3} + x_{d4} + x_{d5} \geq 6$$

$$\sum_{n=1}^{7} x_{dn} \geq 38$$

$$x_{dn} \geq 0, x_{dn} \in Z$$

Where minG is the minimum number of standby traffic police, the x_{dn} (n=1...7) denotes the number of standby police from Monday to Sunday, respectively.

Results and Discussion

Table.6 Results of IP model by Lingo

Variables	Values
x_{d1}	32
x_{d2}	0
x_{d3}	6
x_{d4}	0
x_{d5}	0
x_{d6}	0
x_{d7}	0

Formula (12) was calculated by Lingo, and the objective value is equal to 38. Variable values in IP model are shown in Table 6. In general, a total of 54 traffic policemen are needed for BCIA at least, with 16 of them on road. They are divided into eight teams, that means c_j is equal to eight. Of the 54 traffic police, 38 are on standby, and most of them often handle some paperwork unless there are insufficient police patrols. Thirty-two of standby police begin to work on Monday every week, and the other six traffic policemen standby begin to work on Wednesday. Each police standby officer takes on a night shift and has the right to enjoy a two-day vacation every week.

Conclusions

Based on this study, the following comments can be made:

1. By using FTA method, average events occurrence rate may be expressed by a linear equation, which combines the number of traffic accidents, congestion events, and serious violation events with their key importance coefficients. The average events rate reflects the overall traffic situation over a certain period of time to a certain extent.

2. It is generally believed that the number of traffic police calculated by the Queueing model is always lower than the number calculated by the Poisson distribution. However, when the average events rate is very low, this difference between them is not very obvious, and the former may even be higher than the latter in certain area. It is sure that values of c_j corresponding to 'w_q<=0.25' is always smaller than the 'P=0.95's. Therefore, $c_{0.95j}$ is used to represent the sum of the number of patrol and standby police.

3. After obtaining the number of police officers patrolling the road by Queueing modal and the minimum number of police forces by the Poisson distribution model, the number of standby police is easily calculated out. All of variables above will help to establish a constraint equation for the IP model.

Acknowledgements

This research was funded by Beijing Education Science Planning fund (CGA14159). The authors are thankful for its support.

Conflicts of Interest

There are no conflicts of interest.

References

Adler, N., Hakkert, A. S., Raviv, T., & Sher, M. (2014). The traffic police location and schedule assignment problem. Journal of Multi-Criteria Decision Analysis, 21(5-6), 315-333.

Alfares, H. K. (2007). Operator staffing and scheduling for an IT-help call centre. European Journal of Industrial Engineering, 1(4), 414-430.

Bassett, M. (2000). Assigning projects to optimize the utilization of employees' time and expertise. Computers & Chemical Engineering, 24(2-7), 1013-1021.

Basu, M., & Ghosh, D. (1997). Nonlinear Goal Programming Model for the Development of Metropolitan Police Patrol Units. Opsearch, 34(1), 27-42.

Chaiken, J. M., & Dormont, P. (1978). A patrol car allocation model: Background. Management Science, 24(12), 1280-1290.

D'Amico, S. J., Wang, S. J., Batta, R., & Rump, C. M. (2002). A simulated annealing approach to police district design. Computers & Operations Research, 29(6), 667-684.

Green, L. (1984). A multiple dispatch queueing model of police patrol operations. Management Science, 30(6), 653-664.

Green, L., & Kolesar, P. (1984). The feasibility of one-officer patrol in New York City. Management science, 30(8), 964-981.

Green, L., & Kolesar, P. (1989). Testing the validity of a queueing model of police patrol. Management Science, 35(2), 127-148.

Hickman, M. J., Fricas, J., Strom, K. J., & Pope, M. W. (2011). Mapping police stress. Police Quarterly, 14(3), 227-250.

Keskin, B. B., Li, S. R., Steil, D., & Spiller, S. (2012). Analysis of an integrated maximum covering and patrol routing problem. Transportation Research Part E: Logistics and Transportation Review, 48(1), 215-232.

Kolesar, P. J., Rider, K. L., Crabill, T. B., & Walker, W. E. (1975). A queuing-linear programming approach to scheduling police patrol cars. Operations Research, 23(6), 1045-1062.

Kou, C. K. C., & Liu, G. C. L. G. C. (1996, October). An adaptive police duty scheduling system based on machine learning. In 1996 30th Annual International Carnahan Conference on Security Technology (pp. 212-219). IEEE.

Lou, Y., Yin, Y., & Lawphongpanich, S. (2011). Freeway service patrol deployment planning for incident management and congestion mitigation. Transportation Research Part C: Emerging Technologies, 19(2), 283-295.

Mustapar, W. E., Nasir, D. S. M., Nor, N. A. M., & Abas, S. F. S. (2017, November). Goal programming for cyclical auxiliary police scheduling at UiTM Cawangan Perlis. In AIP Conference Proceedings (Vol. 1905, No. 1, p. 040021). AIP Publishing.

Nag, B. (2014). A MIP model for scheduling India's General elections and police movement. Opsearch, 51(4), 562-576.

Ozbay, K., Iyigun, C., Baykal-Gursoy, M., & Xiao, W. (2013). Probabilistic programming models for traffic *incident management operations planning. Annals of Operations Research, 203(1), 389-406.*

Pal, B. B., Kumar, M., & Sen, S. (2009, December). A linear fuzzy goal programming approach for solving patrol manpower deployment planning problems—A case study. In 2009 International Conference on Industrial and Information Systems (ICIIS) (pp. 244-249). IEEE.

Pal, B. B., Chakraborti, D., Biswas, P., & Mukhopadhyay, A. (2012). An application of genetic algorithm method for solving patrol manpower deployment problems through fuzzy goal programming in traffic management system: a case study. International Journal of Bio-Inspired Computation, 4(1), 47-60.

Sharma, D. K., Ghosh, D., & Gaur, A. (2007). Lexicographic goal programming model for police patrol cars deployment in metropolitan cities. International journal of information and management sciences, 18(2), 173.

Todovic, D., Makajic-Nikolic, D., Kostic-Stankovic, M., & Martic, M. (2015). Police officer scheduling using goal programming. Policing: An International Journal of Police Strategies & Management, 38(2), 295-313.

Van den Bergh, J., Beliën, J., De Bruecker, P., Demeulemeester, E., & De Boeck, L. (2013). Personnel scheduling: A literature review. European Journal of Operational Research, 226(3), 367-385.

Vila, B. (2006). Impact of long work hours on police officers and the communities they serve. American journal of industrial medicine, 49(11), 972-980.

Vila, B., Morrison, G. B., & Kenney, D. J. (2002). Improving shift schedule and work-hour policies and practices to increase police officer performance, health, and safety. Police quarterly, 5(1), 4-24.

Violanti, J. M., Fekedulegn, D., Andrew, M. E., Charles, L. E., Hartley, T. A., Vila, B., & Burchfiel, C. M. (2012). Shift work and the incidence of injury among police officers. American journal of industrial medicine, 55(3), 217-227.

Wu, J. S., & Lou, T. C. (2014). Improving highway accident management through patrol beat scheduling. Policing: An International Journal of Police Strategies & Management, 37(1), 108-125.

Wu, M. C., & Sun, S. H. (2006). A project scheduling and staff assignment model considering learning effect. The International Journal of Advanced Manufacturing Technology, 28(11-12), 1190-1195.

Yanan, D. , & Huayu, F. . (2012). Genetic annealing algorithm for police officer scheduling problem. Computer Engineering and Applications, 48(28), 225-228.

Yin, Y. (2008). A scenario-based model for fleet allocation of freeway service patrols. Networks and Spatial Economics, 8(4), 407-417.

Zhang, Y., & Brown, D. (2014, April). Simulation optimization of police patrol district design using an adjusted simulated annealing approach. In Proceedings of the Symposium on Theory of Modeling & Simulation-DEVS Integrative (p. 18). Society for Computer Simulation International.